Lever mechanism for vibration isolation

Kuinian Li, Mitchell Gohnert
School of Civil and Environmental Engineering,
University of the Witwatersrand, Johannesburg, South Africa,
e-mails: kuinian.li@wits.ac.za, mitchell.gohnert@wits.ac.za

By introducing lever mechanism into the conventional vibration isolation system, new vibration isolation systems such as lever-damper isolator (LDI), lever-spring isolator (LSI) and lever-spring-mass isolator (LSMI) can be developed. The transmissibility of LDI, transmissibility of LSI and that of LSMI are obtained analytically. With numerical simulation, the vibration isolation performance of these systems and the effect of parameters on the performance are investigated. The results show that the performance of traditional vibration isolator can be improved by the introducing of lever mechanism. The results also show that the new systems have less constraint in design.

Key words: Vibration isolator, transmissibility, lever mechanism.

Introduction

There are many cases where vibration control is required, both for improving the performance of machines and equipments, such as automobiles, heavy machines, machine tools, spacecraft launch vehicles, and for making the measuring equipment or navigation equipment work properly in an vibration environment. For vibration control, numerous methods have been employed or proposed. All these vibration control approaches can be categorized passive and active vibration control systems. The two very common passive vibration control approaches are vibration isolation, insertion of a vibration isolator between the source and the receiver of vibration, and vibration absorption, attachment of a vibration absorber (a secondary system) to the primary system. The objective of vibration isolation is to reduce the vibration transmission from an excitation base (source) to the primary system (receiver), a system to be protected. This is typically accomplished by the addition of a suspension, including spring and damper elements, typical components of a conventional vibration isolator.

Although conventional vibration isolator has been widely used in engineering practice, there are inherent limitations on the amount of isolation it can achieve. And the design of a conventional vibration isolator involves trade-off of resonance response, static displacement and high-frequency attenuation. In other words, the design of a passive isolator is the result of compromising of many different or contradictory requirements.

In the ideal case of a mass $m$ supported by a linear spring of stiffness $k$ on a rigid foundation, isolation does not occur until a frequency of $\frac{1}{\sqrt{km}}$. It is evident that a smaller stiffness of the spring component results in a wider frequency range of isolation. However, the load capacity and static stability restrict the stiffness of the spring used to be too small. To overcome this limitation, sometimes the object that is to be protected is fastened to a massive inertia block, which is in turn supported by stiff isolator (Crede, 1951). However, this bypass violates mass constraint for most applications and it will increase the static displacement. Another measure taken to overcome this limitation is the using of nonlinear springs to obtain a high static stiffness and hence a small static displacement, and a small dynamic stiffness, which results in a low natural frequency. The so-called quasi-zero-stiffness (QZS) mechanism (Zhang et al., 2004) is an example of such an approach. QZS mechanisms are generally achieved by combine a negative stiffness element with a positive stiffness element. Although a number of configurations have been proposed to create the negative stiffness effect by making use of spring orientation or buckling, the implementation of QZS mechanism is still complicated for many engineering applications.

The other inherent limitation of conventional vibration isolator arises from the conflictive demand on damping. To achieve good resonance attenuation, high damping is required whilst good high-frequency attenuation needs the damping of the conventional vibration isolator as low as possible. Therefore, with a conventional vibration isolator, good resonance attenuation can only be achieved at the sacrificing of high-frequency attenuation or vice versa.
A solution, referred to as a skyhook vibration isolator, provides damping at resonance without increasing transmissibility at high-frequencies (Karnopp et al., 1974). However, skyhook vibration isolator requires that a viscous damper be connected to an initial reference frame, the “sky”, which is not practical in most situations. In case an initial reference frame is available, very high values of isolator damping is required to achieve a better vibration control result, for there is little difference between the transmissibility of a traditional isolator and that of skyhook isolator when the damping ratio is less than 0.1.

There has been work to realize a skyhook vibration isolator actively by means of sensor, actuator and control electronics (Krasnicki, 1981). There are also other realizations of skyhook isolator using semi-active approaches. While these studies have yield promising results, they usually introduce the need for power source and necessarily add complexity to the isolation system.

A virtual skyhook vibration isolator is proposed by Griffin et al. (2002). Instead of using inertial reference frame, the virtual skyhook vibration isolator uses a secondary tuned spring-mass-damper system. A virtual skyhook can get better high-frequency attenuation, though the resonant response is not attenuated so well as a skyhook isolator.

Due to strict requirements on stiffness and mass of the isolators used in the aerospace industry, Flamnelly (1967) developed a new kind of vibration isolator, dynamic anti-resonance vibration isolator (DAVI), by making use of a levered mass spring combination to generate an anti-resonance frequency in the system. To the author’s knowledge, this can be seen as the first application of lever mechanism in vibration isolation. The lever in a DAVI amplifies the motion of a small mass, which in turn generates large inertial forces. And this inertial amplification can be achieved by various methods, such as using flywheel (Rivin, 2003), hydraulic leverage (Goodwin, 1980) and so on. The basic operation principle of these systems is the same and can be denoted as lever-mass isolator (LMI). Based on this, multi-stage lever type anti-resonant vibration isolator is studied (Yilmaz and Kikuchi, 2006). However, it still belongs to lever-mass isolator.

In this paper, lever-damper isolator (LDI), lever-spring isolator (LSI) and lever-spring-mass isolator (LSMI) are suggested. The transmissibility of LDI, LSI and that of LSMI are formulated. Numerical simulations are performed and the performance of LSI, LDI and LSMI are studied and compared with the performance of a conventional vibration isolator and that of a skyhook. The aim is to show the limitations and potentials of these vibration isolation systems.

**Modeling and Analysis**

**Lever-damper isolator (LDI)**

Introducing lever mechanism in to conventional vibration isolator (CVI), different vibration isolators can be developed. A simplest model of a lever-damper isolator (LDI) is the SDOF system shown diagrammatically in Figure 1.
For the sake of clarity, simple lever is used to model leverage in the isolator. Assuming that the lever rod is massless and rigid; the spring is linear, massless and undamped, then, the system is single-degree of freedom (SDOF) system. Moreover, assuming that the oscillations are small, then linear theory is applicable. The equation of motion is:

\[ m\ddot{x}(t) + c\dot{x}(t) + kx(t) = cL^2\ddot{z}(t) + k\dot{z}(t) \]  

Where \( x(t) \) is the motion of the mass, \( z(t) \) is the motion of the base, \( c \) is the damping coefficient and \( k \) is the mounting stiffness. \( L = L/1 \) is the lever ratio. The displacement transmissibility of the isolator, denoted as \( TR_{L,DD} \), can be obtained by taking the Laplace transform of Equation (1) and solving for the amplitude ratio \( \frac{X(s)}{Z(s)} \), which yields:

\[ TR_{LDI} = \left| \frac{X(s)}{Z(s)} \right| = \frac{1 + 2jxL^2r}{1 - r^2 + 2jxL^2r} \]  

Figure 2 presents the transmissibility of conventional isolator, skyhook isolator and that of a lever-damper isolator (LDI) with lever ratio equals 2, all at the damping ratio of 0.1. It can be seen that
the performance of a skyhook isolator is almost the same as that of a conventional isolator, except that skyhook isolator showed a little bit better attenuation for higher frequencies. Actually, if the damping ratio is below 0.1, the performance of skyhook isolator is almost the same as that of the conventional isolator. However, at the same damping ratio of 0.1, Lever damper isolator showed significant attenuation on resonance, though its performance in higher frequencies attention is not as good as skyhook isolator or conventional isolator. This means that a LDI is more desirable for the resonance attenuation of a low-damped system.

When the lever ratio $L=1$, Equation (2) is deduced to the transmissibility of a conventional vibration isolator.

Figure 3 gives the transmissibility of traditional isolator and that of skyhook isolator, both at a damping ratio of 0.1, and that of a LDI at damping ratio of 0.05. The lever ratio of LDI is set at $L=2$. It can be noticed that compared with traditional isolator or skyhook, LDI can achieve a significant attenuation on resonance with little sacrificing on higher frequencies attention.

It should be pointed out that by introducing lever mechanism another type of lever damper isolator, as shown in Figure 4, can be constructed and its transmissibility can be derived and studied, though the result will not be presented here for space saving.

**Lever-spring isolator (LSI)**

A model of a lever-spring isolator is the SDOF system shown diagrammatically in Figure 5. The equation of motion can be obtained as:

$$m\ddot{x}(t) + c\dot{x}(t) + kL^2 x(t) = c\ddot{z}(t) + kL^2 z(t) \quad (3)$$

Taking the same procedures as that in 1, the transmissibility of this system can be obtained as:

$$TR_{LSI} = \frac{X(s)}{Z(s)} = \frac{L^2 + 2jxr}{L^2 - r^2 + 2jxr} \quad (4)$$

Letting the lever ratio $L$ in Equation 4 equals 1, one will get the transmissibility of a conventional vibration isolator. The performance of LSI and effect of different parameters is studied by numerical simulation.

**Effect of lever ratio $L$**

The damping ration $\varsigma$ is set at 0.1 for both LSI and the traditional vibration isolator. Figure 6 gives the transmissibility of conventional isolator and the transmissibility of LSI, at lever ratio $L = 0.25$, $L = 0.5$, $L = 2$ and $L = 4$ respectively.

Figure 6 demonstrates that with LSI the resonance occurs at frequency ratio $r=L$, rather that at $r=1$ as a conventional vibration isolator does, i.e., LSI moves the resonance from frequency ratio $r=1$ to $r=L$. And, thus, when $L<1$, the resonance frequency can be decreased and the isolator is capable of operating in a lower-frequency range. And the transmissibility over the whole frequency range becomes lower. This is not only very useful for releasing constraint on spring stiffness and mass in passive isolator design, but also produce significant isolation effect over a wider frequency range. For example, when $L=0.5$, at the same damping ration, it can be seen from Figure 6 that, compared with conventional vibration isolator, LSI can achieve much better isolation over the whole frequency range.
The lever ratio of an LSI is set at $L=0.5$. Figure 7 presents the transmissibility of conventional vibration isolator and that of an LSI at different damping ratio. It can be seen that at different damping level, compared with conventional vibration isolator, LSI can always achieve a better isolation effect over whole frequency range.
Lever-mass isolator (LMI), which is also called dynamic anti-resonant vibration isolator (DAVI) as shown diagrammatically in Figure 8, has been studied and applied in practice. The lever in a LMI amplifies the motion of a small mass, which in turn produce large inertial force and, thus, generate an anti-resonant frequency in the system. Anti-resonance occurs when the inertial force generated by the levered mass cancels the spring force.

**Lever-spring-mass isolator (LSMI)**

Combining lever-spring isolator and lever-mass isolator, a lever-spring-mass isolator (LSMI) is suggested as shown in Figure 9.

Equation of motion:

\[ M\ddot{x} = c(\ddot{\xi} - \dot{\dot{x}}) + mL(\ddot{z} - \dot{\dot{z}}) + kL^2(\ddot{z} - x) \]  

Equation(5) can be rewritten as:

\[ (1 + \mu L^2)\ddot{x} + \frac{c}{M}\dot{x} + \frac{k}{M}L^2 x = \mu L(L + 1)\ddot{z} + \frac{c}{M}\dot{z} + \frac{k}{M}L^2 \dot{z} \]  

The transmissibility of LSMI can be obtained from Equation(6) as:

\[ TR_{LSMI} = \frac{L^2 - \mu L(L + 1)r^2 + 2j\xi r}{L^2 - (1 + \mu L^2)r^2 + 2j\xi r} \]  

Where \( \mu = \frac{m}{M} \) is defined as mass ratio of the system.

When \( m = 0 \), i.e., \( \mu = 0 \), Equation(7) becomes:

\[ TR_{LSMI}\big|_{\mu=0} = \frac{L^2 + 2j\xi r}{L^2 - r^2 + 2j\xi r} = TR_{LSI} \]  

That means in this case LSMI degenerates to a LSI.

Furthermore, if \( \mu = 0 \) and \( L=1 \), Equation (7) becomes:

\[ TR_{LSMI}\big|_{\mu=0,L=1} = \frac{1 + 2j\xi r}{1 - r^2 + 2j\xi r} = TR_{CVI} \]  

Equation (9) means that in this case LSMI degenerates to a conventional vibration isolator. It can be seen that LSMI is a mixture of LSI and LMI. Therefore, it should inherit some characteristics of both LSI and LMI, Which is proven by the following numerical simulations.

**Effect of damping ratio**

Figure 10 presents the transmissibility of a conventional vibration isolator, a skyhook isolator and
an LSMI (lever ratio \(L=0.5\) and mass ratio \(\mu = 0.05\)) the following damping ratio \(\zeta = 0.05, 0.1, 0.2\) and 0.9 respectively.

Comparing Figure 10 with Figure 7, it can be seen that similar to LSI, when the lever ratio \(L\) is set at \(L<1\), LSMI can also decrease the resonance frequency and the isolator can operate in a lower frequency range. On the other hand, unlike LSI which can achieve lower response on the whole frequency range than conventional vibration isolator, response of LSMI could be higher than that of skyhook isolator or conventional vibration isolator, though the resonance frequency of an LSMI could be further decreased than LSI by the increasing of effective mass from the levered mass. It can also be seen from Figure 10 that when the damping of the system is not very high it is possible for LSMI to achieve more efficient vibration isolation than a conventional isolator or skyhook isolator at lower frequency range. However, when the damping ratio is very high, say 0.9, there is no significant difference between the isolation effects of an LSMI and that of a conventional isolator; while in this case, a skyhook works extremely well.

**Figure 10. Effect of damping ratio**

![Graphs showing effect of damping ratio](image)

**Effect of mass ratio**

To study the effect of mass ratio of a LSMI, the damping ratio of conventional vibration isolator, skyhook and LSMI are all set at 0.1. The lever ratio of LSMI is set at 0.5. The mass ratio of LSMI takes 0.01, 0.05, 0.1 and 0.2 respectively. Figure 11 presents the simulation results.

Figure 11 shows that with a smaller mass ratio, an LSMI can achieve a very good vibration isolation at lower frequency range. For example, when mass ratio equals to 0.01, an LSMI can achieve a much better isolation result than conventional isolator or even skyhook in the frequency range of \(0 \leq r \leq 5\). When mass ratio equals 0.05, an LSMI can achieve better resonance attenuation than a skyhook in the frequency range up to \(r \leq 5\) and better isolation than a conventional isolator until \(r \leq 4.5\). As the mass ratio goes higher, the attenuation of an LSMI over resonance becomes more significant while attenuation on higher frequency range becomes poor.

Making use of lever mechanism, other types of vibration isolator can be developed. For example, combining LDI with LMI one will form a lever-damper-mass isolator. For saving space, these will not be discussed here. It is worthy to point out that lever mechanism can also be possibly used to modify dynamic vibration absorber for various purposes, though it will not be dealt with here.
**Conclusion**

Employing leverage three new vibration isolators, lever-damper isolator (LDI), lever-spring isolator (LSI) and lever-spring-mass isolator (LSMI) are developed and their performance (transmissibility) and effect of various parameters on the performance are investigated by numerical simulation. The performances of these new isolators are compared with that of a conventional vibration isolator and that of a skyhook. The results show that introducing lever mechanism into conventional vibration isolation system can not only improve the vibration isolation performance of conventional isolator but also release constraints on the design of vibration isolator and, therefore, shows bright prospect for engineering application.

**References**


