

# De Sitter Bubble as a Model of the Observable Universe

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**Abstract:** Schwarzschild's metric of the space inside a sphere of incompressible liquid is taken under focus. We consider a particular case of the metric, where the surface of the liquid sphere meets the radius of gravitational collapse calculated for the mass. It is shown that, in this case, Schwarzschild's metric transforms into de Sitter's metric given that the cosmological  $\lambda$ -term of de Sitter's metric is positive (physical vacuum has positive density). Hence, in the state of gravitational collapse, the  $\lambda$ -field (physical vacuum) is equivalent to an ideal incompressible liquid whose density and pressure satisfy the equation of inflation (noting that positive density yields negative pressure). This result is then applied to the Universe as a whole, because it has mass, density, and radius such as those of a collapsar. The main conclusion of this study is: the Universe is a collapsar, whose internal space, being assumed to be a sphere of incompressible liquid, is a de Sitter space with positive density of physical vacuum.

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**§1. Problem statement.** The main task of this study is to apply an extension of Schwarzschild's metric inside a sphere of incompressible liquid to cosmology. In other words, we will consider the Universe as a sphere of incompressible liquid. The extended Schwarzschild metric was obtained in my previous study [1]. It differs from the classical metric of the space inside a sphere of incompressible liquid, which was introduced in 1916 by Karl Schwarzschild [2], in the term  $g_{11}$  which allows space breaking. Schwarzschild omitted space breaks from consideration, which was a limitation imposed by him on the geometry. In contrast, we consider the geometry per se. This approach has already led us to some success: considering the Sun as a sphere of incompressible liquid, it was

obtained that the break of  $g_{11}$  in the space of the Sun meets the Asteroid strip at the distance of the maximal concentration of substance [1].

The extended Schwarzschild metric has the form [1]

$$ds^2 = \frac{1}{4} \left( 3 \sqrt{1 - \frac{\varkappa \rho_0 a^2}{3}} - \sqrt{1 - \frac{\varkappa \rho_0 r^2}{3}} \right)^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{\varkappa \rho_0 r^2}{3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1.1)$$

where  $\varkappa$  is Einstein's gravitational constant,  $\rho_0$  is the density of the liquid,  $a$  is the radius of the liquid sphere, and  $r$  is the radial coordinate (whose origin is located at the centre of the sphere).

As was shown [1], the internal metric (1.1) of the liquid sphere being expressed through the density  $\rho_0 = \frac{M}{V}$ , the volume  $V = \frac{4\pi a^3}{3}$ , Einstein's constant  $\varkappa = \frac{8\pi G}{c^2}$ , and the Hilbert radius\*  $r_g = \frac{2GM}{c^2}$ , takes the form

$$ds^2 = \frac{1}{4} \left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}} \right)^2 c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2 r_g}{a^3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2). \quad (1.2)$$

Assuming  $r_g = a$  in the formula, we trivially arrive at the metric

$$ds^2 = \frac{1}{4} \left( 1 - \frac{r^2}{a^2} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r^2}{a^2}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1.3)$$

which is a particular case of de Sitter's metric. After the transformation of the time coordinate  $\tilde{t} = \frac{1}{2} t$ , this metric transforms into de Sitter's classical metric

$$ds^2 = \left( 1 - \frac{\lambda r^2}{3} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{\lambda r^2}{3}} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (1.4)$$

where  $\lambda = \frac{3}{a^2} > 0$  in the particular case.

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\*This is the radius at which the field of a massive sphere (approximated as its centre of gravity — a mass-point) is in the state of gravitational collapse ( $g_{00} = 0$ ). It is also known as the Schwarzschild radius, despite the fact that Karl Schwarzschild (1873–1916) never considered gravitational collapse in his papers of 1916 [2,3]. I refer to it as the *Hilbert radius* after David Hilbert (1862–1944) who considered it in 1917 [4], on the basis of the Schwarzschild mass-point solution [3].

Schwarzschild's metric inside a sphere of incompressible liquid is a solution of Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\varkappa T_{\alpha\beta}, \quad (1.5)$$

containing the energy-momentum tensor  $T_{\alpha\beta}$  of ideal liquid, while the  $\lambda$ -term is assumed to be zero. At the same time, de Sitter's metric is a solution of Einstein's equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \lambda g_{\alpha\beta}, \quad (1.6)$$

where the energy-momentum tensor is zero, while the  $\lambda$ -term is nonzero. Since, as was shown, Schwarzschild's metric can be transformed into a particular case of de Sitter's metric, it would be interesting to find a correspondence between the energy-momentum tensor of incompressible liquid and the  $\lambda$ -term.

Proceeding from the formula for the energy-momentum tensor of ideal (non-viscous) incompressible liquid, we will see that the medium is equivalent to the  $\lambda$ -field (physical vacuum) under a particular condition, where the density and pressure satisfy the inflation equation  $p = -\rho c^2$  (keeping in mind that positive density yields negative pressure).

**§2. A sphere of an incompressible liquid in the state of collapse as a model of the Universe.** Many models of the Universe are known, due to relativistic cosmology, as respective solutions to Einstein's equations. Initially, Albert Einstein believed that only stationary models of the Universe can be derived from the field equations. He therefore suggested a de Sitter space as a possible model of the Universe. This is a spherical space, filled with the  $\lambda$ -field (physical vacuum), and is described by de Sitter's metric. Then Alexander Friedmann proved that Einstein's equations can have non-stationary solutions. He obtained a class of solutions (models), which can be both stationary and non-stationary. The non-stationary Friedmann models can be expanding, compressing, or oscillating; the expanding models arise from a singular state, while the compressing and oscillating models can go through singular states during their evolution. All Friedmann models are homogeneous and isotropic. They are commonly accepted as the basis of the theory of a homogeneous, isotropic universe.

Already in 1966, Kyril Stanyukovich [5] had supposed that our Universe is a collapsar — an object in the state of gravitational collapse. He proceeded from a calculation, according to which an object, having mass and density equal to those of the Metagalaxy, has radius equal to

the Hilbert radius calculated for the mass.

A collapsar means a static solution of Einstein's equations. Therefore, I suggest we go back to Einstein's initial suggestion of a de Sitter space, while taking Stanyukovich's calculation into account. Namely, I will consider the Universe as a collapsed sphere of incompressible liquid, described by the extended Schwarzschild metric (1.2), which can also be represented as a de Sitter space (1.3); thus the liquid gets the properties of physical vacuum ( $\lambda$ -field).

The term "gravitational collapse" is regularly used in connexion to the gravitational field derived from a spherical island of mass located in emptiness (for which Einstein's equations take the form  $R_{\alpha\beta} = 0$ ). The metric attributed to such spaces was introduced in 1916 by Karl Schwarzschild [3]. It is known as the Schwarzschild mass-point metric, or the mass-point metric in short

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2), \quad (2.1)$$

where  $M$  is the island's mass (source of the field), while  $r_g = \frac{2GM}{c^2}$  is the Hilbert radius calculated for the mass  $M$ . Once  $r = r_g$ , the time component of the fundamental metric tensor becomes zero ( $g_{00} = 0$ ): all the region under the surface  $r = r_g$  around the massive island arrives at the state of gravitational collapse. If the island's radius  $r$  meets the surface of gravitational collapse, the island is obviously a collapsar.

The radius  $r_g$  is only defined by the mass of the massive island (source of the field). The radius  $r$  of the massive island itself comes from specific properties of the massive island itself. Therefore, in order for a massive island to be a collapsar, we should determine its properties so that its radius is equal to  $r_g$ . We would like to discover such a case.

Consider the space inside a sphere of incompressible liquid, whose radius is  $a$ . This case, first coined by Schwarzschild [2], arrives from Einstein's equations (1.5), where the energy-momentum tensor is attributed to ideal liquid (whose density is constant,  $\rho = \rho_0 = \text{const}$ )

$$T^{\alpha\beta} = \left(\rho_0 + \frac{p}{c^2}\right) b^\alpha b^\beta - \frac{p}{c^2} g^{\alpha\beta}, \quad (2.2)$$

where  $p$  is the pressure of the liquid, while

$$b^\alpha = \frac{dx^\alpha}{ds}, \quad b_\alpha b^\alpha = 1 \quad (2.3)$$

is the four-dimensional velocity vector, which characterizes the reference frame of an observer. The energy-momentum tensor should satisfy the

conservation law

$$\nabla_{\sigma} T^{\alpha\sigma} = 0, \quad (2.4)$$

where  $\nabla_{\sigma}$  is the symbol for generally covariant differentiation.

Assume, according to Stanyukovich [5], that the Universe is a colapsar. In addition to it, assume that the Universe is a sphere of incompressible ideal liquid, where galaxies play the rôle of molecules. In this case, the space of the Universe should be described by the extended Schwarzschild metric (1.2) with an additional condition

$$g_{00} = 0, \quad (2.5)$$

which points to the state of gravitational collapse. This condition, being applied to the metric (1.2), means that

$$g_{00} = \frac{1}{4} \left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r}{a^3}} \right)^2 = 0. \quad (2.6)$$

It follows from (2.6) that a photometric radial distance  $r = r_c$ , at which the gravitational collapse occurs, is

$$r_c = a \sqrt{9 - \frac{8a}{r_g}}, \quad (2.7)$$

thus  $r_c$  takes real values if the radius  $a$  of the liquid sphere is

$$a < \frac{9}{8} r_g, \quad (2.8)$$

while the radius of gravitational collapse becomes zero ( $r_c = 0$ ) under the condition

$$a = \frac{9}{8} r_g = 1.125 r_g. \quad (2.9)$$

Consider a particular case of (2.7), where the surface of the liquid sphere meets the Hilbert radius. In this case, we have

$$a = r_g \quad (2.10)$$

and, as follows from (2.7), the photometric distance also meets the collapse surface  $r_c = r_g = a$ . Hence:

The internal field of a sphere of incompressible liquid in the state of gravitational collapse is equivalent to the external field of a collapsing mass-point as well.

Now, since Schwarzschild's metric of the space inside a sphere of in-

compressible liquid transforms into de Sitter's metric by the collapse condition and the condition  $\lambda = \frac{3}{a^2}$ , we arrive at the conclusion:

Space inside a sphere of incompressible liquid, which is in the state of gravitational collapse, is described by de Sitter's metric, where the  $\lambda$ -term is  $\lambda = \frac{3}{a^2}$ .

All these can be applied to the Universe as a whole, because it has mass, density, and radius such as those of a collapsar. Therefore,

The Universe is a collapsar, whose internal space, being assumed to be a sphere of incompressible liquid, is a de Sitter space with  $\lambda = \frac{3}{a^2}$  (here  $a$  is the radius of the Universe).

### §3. Physically observable characteristics of a de Sitter space.

Herein, I consider physically observable properties of a de Sitter space, described by the metric (1.3), where  $\lambda = \frac{3}{a^2}$ . I use Zelmanov's mathematical apparatus of chronometric invariants [6–8]: chronometrically invariant quantities, being the respective projections of four-dimensional quantities onto the line of time and the three-dimensional section of an observer, are physically observable in his frame of reference.

According to the theory of chronometric invariants, the gravitational potential  $w$  and the linear velocity  $v_i$  of the rotation of space are

$$w = c^2(1 - \sqrt{g_{00}}), \quad v_i = \frac{cg_{0i}}{\sqrt{g_{00}}}, \quad i = 1, 2, 3. \quad (3.1)$$

In both Schwarzschild's metric and de Sitter's metric, all the  $g_{0i}$  are zero, thus  $v_i = 0$  (such a space does not rotate). Therefore, in these spaces, according to the chr.inv.-definition of the gravitational inertial force  $F_i$  and the angular velocity  $A_{ik}$  of the rotation of space [6–8],

$$A_{ik} = \frac{1}{2} \left( \frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k} \right) + \frac{1}{2c^2} (F_i v_k - F_k v_i) = 0, \quad (3.2)$$

$$F_i = \frac{c^2}{c^2 - w} \left( \frac{\partial w}{\partial x^i} - \frac{\partial v_i}{\partial t} \right) = -\frac{c^2}{2g_{00}} \frac{\partial g_{00}}{\partial x^i}. \quad (3.3)$$

Applying the formula for  $g_{00}$ , which follows from the metric (1.3) of the particular de Sitter space we are considering, we obtain

$$F_1 = \frac{c^2 r}{a^2 - r^2}, \quad F_2 = F_3 = 0, \quad (3.4)$$

where  $a^2 = \frac{3}{\lambda}$ . Since we are considering a region of  $r < a$ ,  $F_1$  is positive. Hence, this is a gravitational inertial force of repulsion.

The chr.inv.-metric tensor, in a case of  $v_i = 0$ , takes the form

$$h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k = -g_{ik}, \quad h^{ik} = -g^{ik}, \quad h_k^i = \delta_k^i, \quad (3.5)$$

where its substantial components for the metric (1.3) are

$$h_{11} = \frac{a^2}{a^2 - r^2}, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2 \theta, \quad (3.6)$$

$$h^{11} = \frac{a^2 - r^2}{a^2}, \quad h^{22} = \frac{1}{r^2}, \quad h^{33} = \frac{1}{r^2 \sin^2 \theta}, \quad (3.7)$$

$$h = \det \|h_{ik}\| = \frac{a^2 r^4 \sin^2 \theta}{a^2 - r^2}. \quad (3.8)$$

According to the chr.inv.-definition of the deformation of space,

$$D_{ik} = \frac{1}{2} \frac{* \partial h_{ik}}{\partial t} = 0, \quad D^{ik} = -\frac{1}{2} \frac{* \partial h^{ik}}{\partial t} = 0, \quad (3.9)$$

where  $\frac{* \partial}{\partial t} = \frac{1}{\sqrt{g_{00}}} \frac{\partial}{\partial t}$  is the chr.inv.-operator of differentiation with respect to time. Hence, such a space is free of deformation.

The chr.inv.-Christoffel symbols of the 1st kind and the 2nd kind

$$\Delta_{ij}^k = h^{km} \Delta_{ij,m} = \frac{1}{2} h^{km} \left( \frac{* \partial h_{im}}{\partial x^j} + \frac{* \partial h_{jm}}{\partial x^i} - \frac{* \partial h_{ij}}{\partial x^m} \right), \quad (3.10)$$

are defined through  $\frac{* \partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{* \partial}{\partial t}$ , which is the chr.inv.-operator of differentiation with respect to the spatial coordinates. Their non-zero components of the metric (1.3) are

$$\Delta_{11,1} = \frac{a^2 r}{(a^2 - r^2)^2}, \quad \Delta_{22,1} = -r, \quad \Delta_{33,1} = -r \sin^2 \theta, \quad (3.11)$$

$$\Delta_{12,2} = r, \quad \Delta_{33,2} = -r^2 \sin \theta \cos \theta, \quad (3.12)$$

$$\Delta_{13,3} = r \sin^2 \theta, \quad \Delta_{23,3} = r^2 \sin \theta \cos \theta, \quad (3.13)$$

$$\Delta_{11}^1 = \frac{r}{a^2 - r^2}, \quad \Delta_{22}^1 = -\frac{(a^2 - r^2)r}{a^2}, \quad \Delta_{33}^1 = -\frac{(a^2 - r^2)r}{a^2} \sin^2 \theta, \quad (3.14)$$

$$\Delta_{12}^2 = \frac{1}{r}, \quad \Delta_{33}^2 = -\sin \theta \cos \theta, \quad (3.15)$$

$$\Delta_{13}^3 = \frac{1}{r}, \quad \Delta_{23}^3 = \cot \theta. \quad (3.16)$$

The chr.inv.-curvature tensor  $C_{lkij}$ ,

$$C_{lkij} = H_{lkij} - \frac{1}{c^2} (2A_{ki}D_{jl} + A_{ij}D_{kl} + A_{jk}D_{il} + A_{kl}D_{ij} + A_{li}D_{jk}), \quad (3.17)$$

which possesses all properties of the Riemann-Christoffel tensor in the spatial section, is determined through the chr.inv.-Schouten tensor

$$H_{lki}^{\dots j} = \frac{{}^*\partial\Delta_{kl}^j}{\partial x^i} - \frac{{}^*\partial\Delta_{il}^j}{\partial x^k} + \Delta_{kl}^m\Delta_{im}^j - \Delta_{il}^m\Delta_{km}^j. \quad (3.18)$$

The contracted form  $C_{lk} = C_{lki}^{\dots i}$  of the chr.inv.-curvature tensor is

$$C_{lk} = H_{lk} - \frac{1}{c^2} (A_{kj}D_l^j + A_{lj}D_k^j + A_{kl}D). \quad (3.19)$$

In the absence of space rotation and deformation, which is specific to both Schwarzschild spaces and de Sitter spaces,  $H_{lkij}$  and  $C_{lkij}$  are the same.

For the metric (1.3), we obtain, according to the definition (3.18), the non-zero components of the chr.inv.-curvature tensor:

$$C_{121}^{\dots 2} = C_{131}^{\dots 3} = -\frac{1}{a^2 - r^2}, \quad C_{232}^{\dots 3} = -\frac{r^2}{a^2}, \quad (3.20)$$

thus, respectively,

$$C_{1212} = -\frac{r^2}{a^2 - r^2}, \quad C_{1313} = -\frac{r^2 \sin^2 \theta}{a^2 - r^2}, \quad C_{2323} = -\frac{r^4 \sin^2 \theta}{a^2}, \quad (3.21)$$

and also, the non-zero components of the contracted tensor:

$$C_{11} = -\frac{2}{a^2 - r^2}, \quad C_{22} = -\frac{C_{33}}{\sin^2 \theta} = -\frac{2r^2}{a^2}. \quad (3.22)$$

As a result, we obtain the chr.inv.-curvature (observable curvature) of the three-dimensional space (spatial section). It is

$$C = -\frac{6}{a^2} = \text{const} < 0, \quad (3.23)$$

so a de Sitter space having the metric (1.3) is a space of constant negative three-dimensional curvature, where the curvature is inversely proportional to the square of the radius of the space.

These are the physically observable characteristics of a de Sitter space, which has the particular metric (1.3), where  $\lambda = \frac{3}{a^2}$ .



**§4. The cosmological  $\lambda$ -field is equivalent to an ideal incompressible liquid in the state of inflation.** When looking for an exact solution of Einstein's equations while taking a given distribution of matter (the energy-momentum tensor) into account, we should solve them in common with the law of conservation (2.4), which determines the distribution. As is known, de Sitter spaces are filled with  $\lambda$ -fields, thus they are described by the particular form (1.6) of Einstein's equations. On the other hand, as was shown earlier, a de Sitter space containing  $\lambda = \frac{3}{a^2}$  is a particular case of a Schwarzschild space inside a sphere of incompressible liquid, wherein Einstein's equations have the form (1.5). Our task here is to find, by solving Einstein's equations and the equations of energy-momentum conservation, how the properties of ideal liquid are linked to the  $\lambda$ -field in this particular case.

We therefore consider the general form

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = -\kappa T_{\alpha\beta} + \lambda g_{\alpha\beta} \quad (4.1)$$

of Einstein's equations, which covers both de Sitter spaces and Schwarzschild spaces.

According to the theory of chronometric invariants [6–8], the energy-momentum tensor has three observable chr.inv.-components (as well as any symmetric tensor of the 2nd rank):

$$\rho = \frac{T_{00}}{g_{00}}, \quad J^i = \frac{c T_0^i}{\sqrt{g_{00}}}, \quad U^{ik} = c^2 T^{ik}, \quad (4.2)$$

where  $\rho$  is the chr.inv.-density of the distributed matter,  $J^i$  is the chr.inv.-vector of the density of the momentum in the medium,  $U^{ik}$  is the chr.inv.-stress tensor.

Assume that the space is filled with an ideal (non-viscous) incompressible ( $\rho = \rho_0 = \text{const}$ ) liquid. In this case, the energy-momentum tensor has the form (2.2), where the density and pressure of the liquid satisfy the equation of state

$$\rho c^2 = -p, \quad (4.3)$$

known as *the state of inflation*. Respectively, we obtain the chr.inv.-components of the energy-momentum tensor (2.2). They are

$$\rho = \rho_0, \quad J^i = 0, \quad U^{ik} = p h^{ik} = -\rho_0 c^2 h^{ik}, \quad (4.4)$$

being derived from (2.2) through the condition

$$b^i = \frac{dx^i}{ds} = 0, \quad i = 1, 2, 3, \quad (4.5)$$

which means that the observer accompanies his references. The first chr.inv.-component,  $\rho = \rho_0$ , means that the liquid is incompressible. The second chr.inv.-component,  $J^i = 0$ , means that the liquid does not contain flows of momentum. The third chr.inv.-component,  $U^{ik} = ph^{ik}$ , means that the observer accompanies the medium. In other words, a regular observer rests with respect to the medium and its flows.

Chr.inv.-projections of Einstein's equations (4.1) has been obtained in the framework of the theory of chronometric invariants [6–8]. They are known as the Einstein chr.inv.-equations

$$\frac{* \partial D}{\partial t} + D_{jl} D^{jl} + A_{jl} A^{lj} + * \nabla_j F^j - \frac{1}{c^2} F_j F^j = -\frac{\varkappa}{2} (\rho c^2 + U) + \lambda c^2, \quad (4.6)$$

$$* \nabla_j (h^{ij} D - D^{ij} - A^{ij}) + \frac{2}{c^2} F_j A^{ij} = \varkappa J^i, \quad (4.7)$$

$$\begin{aligned} \frac{* \partial D_{ik}}{\partial t} - (D_{ij} + A_{ij})(D_k^j + A_k^j) + D D_{ik} + 3A_{ij} A_k^j + \\ + \frac{1}{2} (* \nabla_i F_k + * \nabla_k F_i) - \frac{1}{c^2} F_i F_k - c^2 C_{ik} = \\ = \frac{\varkappa}{2} (\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}) + \lambda c^2 h_{ik}, \end{aligned} \quad (4.8)$$

where  $U = h^{ik} U_{ik}$  is the trace of the chr.inv.-stress tensor  $U_{ik}$ , while  $* \nabla_i$  is the symbol for chr.inv.-differentiation. The chr.inv.-components of the conservation law (2.4) have the form [6–8]

$$\frac{* \partial \rho}{\partial t} + D\rho + \frac{1}{c^2} D_{ij} U^{ij} + * \nabla_i J^i - \frac{2}{c^2} F_i J^i = 0, \quad (4.9)$$

$$\frac{* \partial J^k}{\partial t} + D J^k + 2(D_i^k + A_i^k) J^i + * \nabla_i U^{ik} - \frac{2}{c^2} F_i U^{ik} - \rho F^k = 0. \quad (4.10)$$

Take into account that for the metric (1.3)

$$D_{ik} = 0, \quad A_{ik} = 0, \quad J^i = 0, \quad U_{ik} = ph_{ik}, \quad U = 3p, \quad (4.11)$$

and the inflation state  $\rho c^2 = -p$ . Under these conditions, the Einstein chr.inv.-equations (4.6–4.8) take the form

$$* \nabla_j F^j - \frac{1}{c^2} F_j F^j = -\frac{\varkappa}{2} (\rho_0 c^2 + 3p) + \lambda c^2 = (\varkappa \rho_0 + \lambda) c^2, \quad (4.12)$$

$$\begin{aligned} \frac{1}{2} (* \nabla_i F_k + * \nabla_k F_i) - \frac{1}{c^2} F_i F_k - c^2 C_{ik} = \\ = \frac{\varkappa}{2} (\rho_0 c^2 - p) h_{ik} + \lambda c^2 = (\varkappa \rho_0 + \lambda) c^2. \end{aligned} \quad (4.13)$$

We calculate

$${}^*\nabla_j F^j = {}^*\nabla_1 F^1 = \frac{{}^*\partial F^1}{\partial x^1} + \frac{{}^*\partial \ln \sqrt{h}}{\partial x^1} F^1 = \frac{c^2(3a^2 - 2r^2)}{a^2(a^2 - r^2)}, \quad (4.14)$$

$${}^*\nabla_1 F_1 = \frac{{}^*\partial F_1}{\partial x^1} - \Delta_{11}^1 F_1 = \frac{c^2}{a^2 - r^2} + \frac{c^2 r^2}{(a^2 - r^2)^2}, \quad (4.15)$$

$${}^*\nabla_2 F_2 = -\Delta_{22}^1 F_1 = \frac{c^2 r^2}{a^2}, \quad (4.16)$$

$${}^*\nabla_3 F_3 = -\Delta_{33}^1 F_1 = \frac{c^2 r^2 \sin^2 \theta}{a^2}, \quad (4.17)$$

then substitute these, and also  $F_1$ ,  $C_{11}$ ,  $C_{22}$ ,  $C_{33}$  calculated according to the formulae of §3, into the Einstein chr.inv.-equations (4.12–4.13). After algebra, we obtain that only one equation of the Einstein chr.inv.-equations remains non-vanishing:

$$\frac{3c^2}{a^2} = (\varkappa \rho_0 + \lambda) c^2. \quad (4.18)$$

Consider two formal cases for this equation, satisfying both the Schwarzschild metric and the particular de Sitter metric. Namely:

- 1) A case, where  $T_{\alpha\beta} \neq 0$  and  $\lambda = 0$ . This means that the space is filled only with distributed matter (ideal incompressible liquid, in this case). Thus, we obtain, from the Einstein chr.inv.-equation (4.18), the density and pressure of the liquid

$$\rho_0 = \frac{3}{\varkappa a^2}, \quad p = -\rho_0 c^2 = -\frac{3c^2}{\varkappa a^2} = \text{const}, \quad (4.19)$$

while the chr.inv.-equations of the conservation law (4.9–4.10) are satisfied as identities;

- 2) Another option is that of  $T_{\alpha\beta} = 0$  and  $\lambda \neq 0$ . In this case, the space is filled only with physical vacuum ( $\lambda$ -field). Thus, the Einstein chr.inv.-equation (4.18) reduces to

$$\lambda = \frac{3}{a^2} > 0, \quad (4.20)$$

so the density and pressure of physical vacuum are expressed through the  $\lambda$ -term, according to the chr.inv.-equations of the conservation law (4.9–4.10), as

$$\rho_0 = \frac{\lambda}{\varkappa}, \quad p = -\frac{\lambda c^2}{\varkappa} = \text{const}. \quad (4.21)$$

Therefore, since these two cases meet each other in the particular case under consideration, we arrive at the conclusion:

The  $\lambda$ -field (physical vacuum), which fills a particular de Sitter space, where  $\lambda = \frac{3}{a^2} > 0$ , is equivalent to an ideal incompressible liquid in the state of inflation.

**§5. Physically observable characteristics of a sphere of incompressible liquid.** Here we compare the details of two different states of the space inside a sphere of incompressible liquid:

- 1) A regular state of the liquid sphere, where its radius  $a$  is much larger than the Hilbert radius  $r_g$  calculated for the mass ( $a \gg r_g$ ). I refer to such an object as a *Schwarzschild bubble*, since its internal space is described by the Schwarzschild metric (1.2);
- 2) The liquid sphere is a collapsar — a body in the state of gravitational collapse. In this case, the surface of the sphere meets its Hilbert radius ( $a = r_g$ ). I suggest that such an object should be referred to as a *de Sitter bubble*. This is because its internal space is described by the particular de Sitter metric (1.3).

First of all, we would like to point out numerous principal differences of this consideration from that according to the Schwarzschild mass-point metric utilized by most relativists when considering collapsars [9].

According to the mass-point metric (2.1),  $g_{00} > 0$  in the space outside the collapsed surface ( $r > r_g$ ),  $g_{00} = 0$  on the surface ( $r = r_g$ ), and  $g_{00} < 0$  in the space inside it ( $r < r_g$ ). Thus the signature condition  $g_{00} > 0$  is violated inside gravitational collapsars. In order to restore the signature condition  $g_{00} > 0$  inside collapsars, another metric is suggested: it is derived from the mass-point metric (2.1) by substitution of  $r = c\tilde{t}$  and  $ct = \tilde{r}$ , thus space and time replace each other. As a result, the signature condition remains valid inside collapsars, but is violated in the regular space surrounding them [9]. Also, the mass-point metric does not specify the body's radius. In other words, we cannot recognize, without additional conditions, whether the object is a collapsar, or not.

By contrast, the signature condition  $g_{00} > 0$  is satisfied everywhere inside a collapsar filled with incompressible liquid (Schwarzschild space) or physical vacuum (de Sitter's space). In addition to it, both metrics contain the radius of space. Thus, we can clearly recognize, from the metric itself, that the considered object is a collapsar ( $a = r_g$ ). These are advantages of our approach.

Probably, there are many such objects in the Universe: consisting of a substance similar to ideal incompressible liquid, they may trans-

form into collapsars at the final stage of their evolution, thus becoming de Sitter bubbles. These objects are hidden from observation, because, being collapsars, they never allow light to leave their internal space for the cosmos.

Let us derive a formula for the chr.inv.-vector of the gravitational inertial force from the metric (1.2). We obtain that just one (radial) component of the force is non-zero. It is

$$F_1 = -\frac{c^2 r_g r}{a^3} \frac{1}{\left(3\sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}}\right) \sqrt{1 - \frac{r_g r^2}{a^3}}}. \quad (5.1)$$

Since  $r < a$  inside the sphere,  $F_1 < 0$  therein. Hence, this is a force of attraction. It is  $F_1 = 0$  at the centre of the sphere, and  $F_1 \rightarrow -\infty$  on its surface (the surface of gravitational collapse).

Consider a regular case, where  $a \gg r_g$ . Expanding  $\sqrt{1 - \frac{r_g r^2}{a^3}}$  into series, while neglecting the high order terms, we obtain

$$\sqrt{1 - \frac{r_g r^2}{a^3}} \approx 1 - \frac{r_g r^2}{2a^3}, \quad (5.2)$$

thus, once  $r = a$ , we have

$$\sqrt{1 - \frac{r_g}{a}} \approx 1 - \frac{r_g}{2a}. \quad (5.3)$$

Substituting (5.2) into (5.1), we obtain

$$F_1 \approx -\frac{c^2 r_g r}{2a^3} = -\frac{GM r}{a^3}. \quad (5.4)$$

If  $r = a$ , we obtain a Newtonian gravitational force of attraction, which is  $F_1 \approx -\frac{GM}{a^2}$ .

It is easy to show that  $F_1$  (5.1) by  $a = r_g$  takes the form

$$F_1 = \frac{c^2 r}{a^2 - r^2} > 0, \quad (5.5)$$

which is a non-Newtonian gravitational force of repulsion:

The gravitational inertial force inside a regular sphere of incompressible liquid and in that in the state of being a collapsar has opposite signs. In a regular liquid sphere (Schwarzschild bubble), this is a Newtonian gravitational force of attraction. In a liquid sphere which is a collapsar (de Sitter bubble), this is a repulsing non-Newtonian gravitational force.

The pressure inside a regular liquid sphere (1.2) is formulated as [1]

$$p = \rho_0 c^2 \frac{\sqrt{1 - \frac{r_g r^2}{a^3}} - \sqrt{1 - \frac{r_g}{a}}}{3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}}}, \quad (5.6)$$

so  $p > 0$  under  $a \gg r_g$ . Once  $a = r_g$ , the pressure takes the form

$$p = -\rho_0 c^2 = \text{const}, \quad (5.7)$$

thus the medium is in the state of inflation. Since  $\rho_0 > 0$ , we obtain that  $p < 0$  inside de Sitter bubbles. So, we conclude:

The pressure is positive in a regular sphere of incompressible liquid. It is negative in a liquid sphere, which is a collapsar.

Consider the chr.inv.-curvature tensor  $C_{lkij}$  for the metric (1.2). First, we obtain the components of the chr.inv.-metric tensor

$$h_{11} = \frac{1}{1 - \frac{r_g r^2}{a^3}}, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2 \theta, \quad (5.8)$$

$$h^{11} = 1 - \frac{r_g r^2}{a^3}, \quad h^{22} = \frac{1}{r^2}, \quad h^{33} = \frac{1}{r^2 \sin^2 \theta}, \quad (5.9)$$

$$h = \det \|h_{ik}\| = \frac{r^4 \sin^2 \theta}{1 - \frac{r_g r^2}{a^3}}, \quad (5.10)$$

and the chr.inv.-Christoffel symbols

$$\Delta_{11,1} = \frac{r_g r}{a^3} \frac{1}{\left(1 - \frac{r_g r^2}{a^3}\right)^2}, \quad \Delta_{22,1} = -r, \quad \Delta_{33,1} = -r \sin^2 \theta, \quad (5.11)$$

$$\Delta_{12,2} = r, \quad \Delta_{33,2} = -r^2 \sin \theta \cos \theta, \quad (5.12)$$

$$\Delta_{13,3} = r \sin^2 \theta, \quad \Delta_{23,3} = r^2 \sin \theta \cos \theta, \quad (5.13)$$

$$\left. \begin{aligned} \Delta_{11}^1 &= \frac{r_g r}{a^3 \left(1 - \frac{r_g r^2}{a^3}\right)}, & \Delta_{22}^1 &= -r \left(1 - \frac{r_g r^2}{a^3}\right) \\ \Delta_{33}^1 &= -r \left(1 - \frac{r_g r^2}{a^3}\right) \sin^2 \theta \end{aligned} \right\}, \quad (5.14)$$

$$\Delta_{12}^2 = \frac{1}{r}, \quad \Delta_{33}^2 = -\sin \theta \cos \theta, \quad (5.15)$$

$$\Delta_{13}^3 = \frac{1}{r}, \quad \Delta_{23}^3 = \cot \theta. \quad (5.16)$$

Then we obtain the non-zero components of  $C_{iklj}$

$$C_{1212} = \frac{C_{1313}}{\sin^2\theta} = -\frac{r_g}{a^3} \frac{r^2}{1 - \frac{r_g r^2}{a^3}}, \quad C_{2323} = -\frac{r_g}{a^3} r^4 \sin^2\theta, \quad (5.17)$$

which coincide with those (3.21) obtained for the particular de Sitter metric (1.3) by the condition  $a = r_g$ , i.e. when the liquid sphere is a col-lapsar. Contracting these with  $h_{ik}$ , we obtain the non-zero components of the contracted chr.inv.-curvature tensor

$$C_{11} = -\frac{2r_g}{a^3} \frac{1}{1 - \frac{r_g r^2}{a^3}}, \quad C_{22} = \frac{C_{33}}{\sin^2\theta} = -\frac{2r_g r^2}{a^3}, \quad (5.18)$$

and also the chr.inv.-curvature scalar (observable curvature of the three-dimensional space)

$$C = -\frac{6r_g}{a^3} = \text{const} < 0, \quad (5.19)$$

which coincides, by the collapse condition  $a = r_g$ , with the respective values (3.22) and (3.23), obtained for the particular de Sitter metric (1.3). Hence, a Schwarzschild space with the metric (1.2) has a constant negative observable (three-dimensional) curvature space.

It should be noted that, as one may find in any textbook of the theory of relativity and relativistic cosmology, de Sitter spaces are constant curvature spaces, while Schwarzschild spaces are not. This commonly accepted terminology is based on the four-dimensional curvature  $K$ . The observable (three-dimensional) chr.inv.-curvature  $C$  is calculated in another way; it is linked to  $K$  only in constant curvature spaces such as de Sitter spaces (see §5.3 in [10], for details). Thus,

In a de Sitter space, the four-dimensional curvature  $K$  and observable (three-dimensional) curvature  $C$  are constants. A Schwarzschild space, which is inside a sphere of incompressible liquid, has a variable four-dimensional curvature  $K$  and a constant observable (three-dimensional) curvature  $C$ .

The observable three-dimensional curvature of such a Schwarzschild bubble has a radius  $\mathfrak{R}$ , which, coming from the relation

$$C = -\frac{6r_g}{a^3} = \frac{1}{\mathfrak{R}^2}, \quad (5.20)$$

which is obvious for a liquid sphere, is imaginary

$$\mathfrak{R} = \frac{ia\sqrt{a}}{\sqrt{6r_g}}. \quad (5.21)$$

Respectively, the observable curvature radius of a de Sitter bubble, according to the formula of  $C$  (3.23), is imaginary as well

$$\Re = \frac{ia}{\sqrt{6}}. \quad (5.22)$$

Now, we consider the four-dimensional curvature of spaces with the metrics (1.2) and (1.3). The Riemann-Christoffel curvature tensor  $R_{\alpha\beta\gamma\delta}$  has three chr.inv.-components [6–8]

$$X^{ik} = -c^2 \frac{R_{0\cdot 0\cdot}^{i\cdot k}}{g_{00}}, \quad Y^{ijk} = -c \frac{R_{0\cdot\cdot}^{ijk}}{\sqrt{g_{00}}}, \quad Z^{iklj} = c^2 R^{iklj}, \quad (5.23)$$

which, according to the theory of chronometric invariants, are generally formulated through the chr.inv.-characteristics of the space of reference of an observer as follows (the indices in  $X^{ik}$ ,  $Y^{ijk}$ ,  $Z^{iklj}$  have been lowered here by the chr.inv.-metric tensor  $h_{ik}$ ):

$$X_{ik} = \frac{{}^* \partial D_{ik}}{\partial t} - (D_i^l + A_i^l)(D_{kl} + A_{kl}) + \frac{1}{2} ({}^* \nabla_i F_k + {}^* \nabla_k F_i) - \frac{1}{c^2} F_i F_k, \quad (5.24)$$

$$Y_{ijk} = {}^* \nabla_i (D_{jk} + A_{jk}) - {}^* \nabla_j (D_{ik} + A_{ik}) + \frac{2}{c^2} A_{ij} F_k, \quad (5.25)$$

$$Z_{iklj} = D_{ik} D_{lj} - D_{il} D_{kj} + A_{ik} A_{lj} - A_{il} A_{kj} + 2A_{ij} A_{kl} - c^2 C_{iklj}. \quad (5.26)$$

Because  $A_{ik} = 0$  and  $D_{ik} = 0$  for both the metric (1.2) and the metric (1.3), the formulae (5.24–5.26) take a simplified form, which is

$$X_{ik} = \frac{1}{2} ({}^* \nabla_i F_k + {}^* \nabla_k F_i) - \frac{1}{c^2} F_i F_k, \quad (5.27)$$

$$Y_{ijk} = 0, \quad (5.28)$$

$$Z_{iklj} = -c^2 C_{iklj}. \quad (5.29)$$

In particular, we see that, in the metrics (1.2) and (1.3) (that is, in the space inside a Schwarzschild bubble or a de Sitter bubble respectively), the spatial observable projection  $Z_{iklj}$  of the Riemann-Christoffel curvature tensor (its distribution along the three-dimensional spatial section) is proportional to the chr.inv.-curvature tensor  $C_{iklj}$ , taken with the opposite sign:

The observable distribution of the Riemann-Christoffel curvature tensor inside both a Schwarzschild bubble and a de Sitter bubble is the same as that of the observable three-dimensional curvature tensor therein, but has the opposite sign.



Let us calculate  $X_{ik}$  for the metric (1.2). This is the chr.inv.-projection of the Riemann-Christoffel curvature tensor onto the line of time of an observer. Its formula (5.27) can be re-written, expanding the symbol of the chr.inv.-differentiation, in the form

$$X_{ik} = \frac{1}{2} \left( \frac{* \partial F_i}{\partial x^k} + \frac{* \partial F_k}{\partial x^i} \right) - \Delta_{ik}^m F_m - \frac{1}{c^2} F_i F_k, \quad (5.30)$$

thus we obtain nonzero components of  $X_{ik}$ . They are

$$X_{11} = -\frac{c^2 r_g}{a^3} \frac{1}{\left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}} \right) \sqrt{1 - \frac{r_g r^2}{a^3}}}, \quad (5.31)$$

$$X_{22} = -\frac{c^2 r_g}{a^3} \frac{r^2 \sqrt{1 - \frac{r_g r^2}{a^3}}}{3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}}}, \quad (5.32)$$

$$X_{33} = -\frac{c^2 r_g}{a^3} \frac{r^2 \sin^2 \theta \sqrt{1 - \frac{r_g r^2}{a^3}}}{3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}}}. \quad (5.33)$$

Assuming  $a = r_g$  that means the metric (1.3), we obtain the same spatial components inside a de Sitter bubble

$$X_{11} = \frac{c^2}{a^2 - r^2}, \quad X_{22} = \frac{c^2 r^2}{a^2}, \quad X_{33} = \frac{c^2 r^2 \sin^2 \theta}{a^2}. \quad (5.34)$$

We see that all non-zero components of  $X_{ik}$  are negative in Schwarzschild bubbles, while they are positive in de Sitter bubbles.

Let us compare the formulae of  $X_{11}$  with the respective formulae of  $F_1$  in Schwarzschild bubbles (5.1) and in de Sitter bubbles (3.4). We see that in both cases they are connected by the relation

$$F_1 = r X_{11}, \quad (5.35)$$

thus we arrive at the following important result:

The time observable component of the Riemann-Christoffel curvature tensor has the same numerical value, but opposite signs in the spaces of a Schwarzschild bubble and a de Sitter bubble. Newtonian gravitational forces of attraction in Schwarzschild bubbles and non-Newtonian gravitational forces of repulsion in de Sitter bubbles are only due to the time observable component of the curvature tensor.

**§6. Conditions of inhomogeneity and anisotropy.** According to the theory of chronometric invariants [6,8], the conditions of homogeneity have the form

$$\left. \begin{aligned} {}^*\nabla_j F_i = 0, \quad {}^*\nabla_j A_{ik} = 0, \quad {}^*\nabla_j D_{ik} = 0, \quad {}^*\nabla_j C_{ik} = 0 \\ \frac{{}^*\partial\rho}{\partial x^i} = 0, \quad \frac{{}^*\partial p}{\partial x^i} = 0, \quad {}^*\nabla_j \beta_{ik} = 0, \quad {}^*\nabla_j q_i = 0 \end{aligned} \right\}, \quad (6.1)$$

where  $\beta_{ik} = \alpha_{ik} - \frac{1}{3}\alpha h_{ik}$  is the anisotropic part of the viscous stress tensor  $\alpha_{ik}$ ,  $\alpha = \alpha_n^n$ , and  $q_i = c^2 J_i$  is the chr.inv.-vector of the density of the flow of energy. In other words, once a three-dimensional spatial section satisfies the conditions (6.1), it is homogeneous from the point of view of an observer located in it. The conditions of isotropy are

$$F_i = 0, \quad A_{ik} = 0, \quad \Pi_{ik} = 0, \quad \Sigma_{ik} = 0, \quad \beta_{ik} = 0, \quad q_i = 0, \quad (6.2)$$

where  $\Pi_{ik} = D_{ik} - \frac{1}{3}Dh_{ik}$  and  $\Sigma_{ik} = C_{ik} - \frac{1}{3}Ch_{ik}$  characterize, respectively, the anisotropy of the deformation and curvature of space. If a spatial section satisfies the conditions (6.2), it is observed as isotropic.

Let us apply the physical conditions of the metrics (1.2) and (1.3) to the conditions of homogeneity and isotropy. In both these metrics,  $A_{ik} = 0$  and  $D_{ik} = 0$ . Also, we should take into account that  $\rho_0 = const$ ,  $\beta_{ik} = 0$ , and  $J_i = 0$  (see previous paragraphs of this paper, for details). As a result, the conditions of homogeneity (6.1) and isotropy (6.2) take a simplified form: the conditions of homogeneity become

$${}^*\nabla_j F_i = 0, \quad {}^*\nabla_j C_{ik} = 0, \quad \frac{{}^*\partial p}{\partial x^i} = 0, \quad (6.3)$$

while the conditions of isotropy become

$$F_i = 0, \quad \Sigma_{ik} = 0. \quad (6.4)$$

Let us calculate  ${}^*\nabla_j C_{ik}$  and  $\Sigma_{ik} = C_{ik} - \frac{1}{3}Ch_{ik}$  for the metrics (1.2) and (1.3), according to the formulae of  $C_{ik}$  obtained in §3 and §5, respectively. We obtain that  ${}^*\nabla_j C_{ik} = 0$  and  $\Sigma_{ik} = 0$  are satisfied in both cases, i.e. in both Schwarzschild bubbles and de Sitter bubbles.

However,  $F_i \neq 0$  in both the metrics (1.2) and (1.3). This means that one of the conditions of isotropy (6.4), namely —  $F_i = 0$ , is violated in Schwarzschild bubbles and de Sitter bubbles.

Two conditions  ${}^*\nabla_j F_i = 0$  and  $\frac{{}^*\partial p}{\partial x^i} = 0$  of the conditions of homogeneity (6.3) remain for consideration.

First, we calculate  ${}^*\nabla_j F_i = \frac{{}^*\partial F_i}{\partial x^j} - \Delta_{ji}^m F_m$  for the metric (1.2). We use the formula for  $F_1$  (5.1), which is the solely non-zero component of

the force. We obtain

$$\begin{aligned} {}^*\nabla_1 F_1 = & -\frac{c^2 r_g}{a^3} \frac{1}{\sqrt{1 - \frac{r_g r^2}{a^3}} \left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}} \right)} + \\ & + \frac{c^2 r_g^2}{a^6} \frac{r^2}{\left( 1 - \frac{r_g r^2}{a^3} \right) \left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}} \right)^2} \neq 0, \end{aligned} \quad (6.5)$$

$${}^*\nabla_2 F_2 = \frac{{}^*\nabla_3 F_3}{\sin^2 \theta} = -\frac{c^2 r_g}{a^3} \frac{r^2 \sqrt{1 - \frac{r_g r^2}{a^3}}}{3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}}} \neq 0. \quad (6.6)$$

For the metric (1.3), we use the formula for  $F_1$  (3.4). We obtain

$${}^*\nabla_1 F_1 = \frac{c^2 a^2}{(a^2 - r^2)^2} \neq 0, \quad {}^*\nabla_2 F_2 = \frac{{}^*\nabla_3 F_3}{\sin^2 \theta} = \frac{c^2 r^2}{a^2} \neq 0 \quad (6.7)$$

(these formulae can also be derived from the previous by substituting the condition  $r_g = a$ ).

We see that the condition  ${}^*\nabla_j F_i = 0$  is violated in both Schwarzschild bubbles and de Sitter bubbles.

Calculating  $\frac{{}^*\partial p}{\partial x^i}$  for the metric (1.2), where the pressure  $p$  is expressed as (5.6), we obtain

$$\frac{\partial p}{\partial r} = -\frac{2r_g r}{a^3} \frac{\rho_0 c^2 \sqrt{1 - \frac{r_g}{a}}}{\left( 3 \sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r_g r^2}{a^3}} \right)^2} \neq 0, \quad (6.8)$$

while for the metric (1.3), where  $p = -\rho_0 c^2 = \text{const}$  (5.7), we have

$$\frac{\partial p}{\partial r} = 0. \quad (6.9)$$

In other words, the condition  $\frac{{}^*\partial p}{\partial x^i} = 0$  is violated in Schwarzschild bubbles, but is satisfied in de Sitter bubbles.

Finally, we conclude:

Space inside Schwarzschild bubbles and de Sitter bubbles is inhomogeneous and anisotropic due to the presence of the gravitational inertial force  $F_i$ . Also, the pressure  $p$  inside a Schwarzschild bubble is a function of distance, which generates an additional effect on the inhomogeneity of space.

At the same time, matter is homogeneously and isotropically distributed therein: this is incompressible liquid, which fills Schwarzschild bubbles, and physical vacuum ( $\lambda$ -field), which fills de Sitter bubbles. This is because the density of the liquid is  $\rho_0 = \text{const}$  in Schwarzschild bubbles (despite  $p \neq \text{const}$  therein), as well as  $\rho_0 = \text{const}$  of physical vacuum (in the state of inflation,  $p = -\rho_0 c^2$ ) in de Sitter bubbles. In brief, this situation can be resumed as follows:

Despite the fact that space inside Schwarzschild bubbles is inhomogeneous and anisotropic, incompressible liquid is distributed homogeneously and isotropically therein. The same is true about de Sitter bubbles (filled with physical vacuum).

A short important note should be made concerning the gravitational inertial force  $F_i$ , which is the main factor of inhomogeneity and anisotropy of Schwarzschild bubbles and de Sitter bubbles.

Consider the space inside a de Sitter bubble. This is a de Sitter space, where the  $\lambda$ -term takes a particular value of  $\lambda = \frac{3}{a^2} > 0$ . In this case, de Sitter's metric takes the form (1.3) and, as was shown in §4, the  $\lambda$ -field has properties of ideal incompressible liquid in the state of inflation. We have already obtained  $F_1$  for the metric (1.3). We calculate the regular (contravariant) vector  $F^1$  of the gravitational inertial force from  $F_1$  (3.4), by lifting the index with the contravariant chr.inv.-metric tensor  $h^{ik}$  (3.7). We obtain

$$F^1 = \frac{c^2 r}{a^2} = \frac{\lambda c^2 r}{3}. \quad (6.10)$$

Hubble's constant  $H = (2.3 \pm 0.3) \times 10^{-18} \text{ sec}^{-1}$  is expressed through the radius of the Universe  $a = 1.3 \times 10^{28} \text{ cm}$  as  $H = \frac{c}{a}$ . Taking this into account, we obtain

$$F^1 = H^2 r, \quad (6.11)$$

where Hubble's constant plays the rôle of a fundamental frequency. This formula meets the result recently obtained by Rabounski [11], according to which the Hubble redshift is due to the rotation of the isotropic space (home of photons) at the velocity of light. As was then shown [12], this effect is presented in any case, even if the non-isotropic space (home of solid bodies) does not rotate or deform.

Thus, according to the formula (6.11), the Hubble redshift has also been explained in the space inside de Sitter bubbles. This is despite that fact that the space does not expand or compress therein (it is free of deformation according to de Sitter's metric), i.e. the de Sitter bubble is a static cosmological model.

**§7. Conclusion.** In conclusion, we have arrived at Einstein's initial suggestion of de Sitter space as the basic cosmological model of our Universe (see page 5). Besides, it has been shown that this model satisfies the observed parameters of the Universe only in a particular case, where it is a collapsar (de Sitter bubble).

Among many advantages of the de Sitter bubble model, which have been elaborated upon in this paper, one of the most important is that the model allows us to calculate the characteristics of the Universe. This is in contrast to the Friedmann models, where, as is known, the parameter  $R(t)$  is indefinite: this is an arbitrary function contained in the metric, so one should introduce it according to physical suggestions, which is not so satisfactory. In the de Sitter bubble model, the parameters of the Universe are unambiguously determined by the metric. All we need to do is substitute  $a = r_g = \frac{2GM}{c^2}$  and the numerical values of the physical constants into the formulae obtained for the model.

For instance, let us substitute  $a = 1.3 \times 10^{28}$  cm, which is the radius of our Universe. We obtain the following characteristics, which characterize the Universe as a de Sitter bubble

$$r_g = \frac{2GM}{c^2} = a = 1.3 \times 10^{28} \text{ cm}, \quad (7.1)$$

$$M = \frac{ac^2}{2G} = 8.8 \times 10^{55} \text{ gram}, \quad (7.2)$$

$$\rho_0 = \frac{3M}{4\pi a^3} = \frac{3c^2}{8\pi G a^2} = 9.5 \times 10^{-30} \text{ gram/cm}^3, \quad (7.3)$$

$$\lambda = \frac{3}{a^2} = \varkappa \rho_0 = 1.8 \times 10^{-56} \text{ cm}^{-2} \quad (7.4)$$

$$p = -\rho_0 c^2 = -\frac{\lambda c^2}{\varkappa} = -\frac{3c^2}{a^2 \varkappa} = 8.6 \times 10^{-9} \text{ dynes/cm}^2. \quad (7.5)$$

These theoretical values correspond to those produced according to observational estimations. Therefore, the de Sitter bubble model suggested here is good enough to be a valid model of the Universe.

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**ABRAHAM ZELMANOV**

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