Peer-reviewed and Open access journal
ISSN: 1804-5006 | www.academicpublishingplatforms.com
The primary version of the journal is the on-line version

BEH - Business and Economic Horizons
Volume 9 | Issue 3 | 2013 | pp. 28-40
DOI: http://dx.doi.org/10.15208/beh.2013.11

Forecasting economic crises using gradient measurement of development and log-logistic function

Rafał Siedlecki [1], ***Daniel Papla*** [2]
[1]*Department of Corporate Finance and Value Management, Wroclaw University of Economics, Poland*
[2] *Department of Financial Investments and Risk Management, Wroclaw University of Economics, Poland*

corresponding e-mail: daniel.papla@ue.wroc.pl
corresponding address: DS Ślężak, Off.20, Komandorska 118/120, Wrocław, Poland

This paper shows that regardless of how good the economic situation is, sooner or later certain difficulties will appear. Hence, the idea has emerged to give the analytical form to the logistic law based gradient measurement (synthetic measure) of selected financial data, which enables forecasting crises and economic downturns. The presented proposal of determining the economic cycle and the model of forecasting using the modified logistic function (log-logistic function) was tested with use of data regarding the Polish economy.

JEL Classifications: C130, C220, C530, E320

Keywords: Law of growth, forecasting, economic crisis, time series analysis, warning signals, S-curve

Introduction

Financial crises are an important phenomenon for the economy because during crisis the cost of intermediation and the cost of credit increase, and moreover access to credit is more difficult. This results in reduced activity in the real property sector, which may lead to a crisis therein.

The quite high incidence of financial crises may lead to the conclusion that the financial sector is particularly sensitive to various types of disturbances. In particular, the recent crisis has shown the sensitivity of the global economy to disturbances in the era of globalization (BIS, 2009; Brunnermeier, 2009; Coffee, 2009; Guillen, 2009; Kolb, 2010; Shiller, 2008). There are certain ways to cope with this problem, see for example (Kurach and Stelmach, 2009; Papla and Siedlecki, 2013).

Economic disturbances have a certain regularity, which permits distinguishing the so-called business cycles and seasonal cycles. It is very difficult to provide a clear and precise definition of the business cycle. The classic definition was given by A. Burns and W. Mitchell (Burns, Mitchell, 1946).

Trajectories of cycles are similar to each other, which allows the division of cycles into phases. As follows from the mentioned definition, A. Burns and W. Mitchell argue that the cycle is divided into four phases (Burns, Mitchell, 1946):

- the recession (crisis) - is characterized by a significant decline in production and an increase in unemployment
- depression - a reduction in the rate of decline in production with a simultaneous increase in employment
- recovery - a growth in production and employment
- expansion (boom) - a high level of production and low unemployment.

- Mintz believes that in the economy "Growth cycles are fluctuations in aggregate economic activity. A growth cycle consists of a period of relatively high growth rates occurring at about the same time in many economic activities, followed by a period of similarly widespread low growth rates which merges into the high-growth phase of the next cycle" (Mintz, 1972). Therefore, there are only two stages in the cycle:
- a relatively high growth rate
- a low growth rate.

Fluctuations in business cycles are often irregular. They have different length and amplitude. This has enabled the development of a number of cycles that describe changes in the economy. This is evident in the time series of economic quantities such as production, employment, income and expenditure of the population, or indicators such as GDP, inflation and money market rates. In order to measure the economic situation, one aggregate indicator characterizing the overall condition of the economy or a company can be used, or a group of indicators describing various components of the economy. It may be noted that an economic boom is characterized by a rising trend of stimulating indicators and a declining trend of destimulating indicators. Important problems are, however, proper selection of indicators, determination of their respective weights and determination of the resultant of changes in individual indicators. As can be noticed, the economic cycle measurement is quite difficult and most frequently one aggregate indicator (e.g. GDP), complemented with other indicators of macroeconomic statistics (e.g., investment, employment), is employed. As follows from the aforementioned definition, forms of fluctuations are not clearly identified in the economic cycle, hence the literature distinguishes a number of cycles.

A modern business cycle in developed countries usually lasts for a period of 2.5-9.5 years[1], predominantly for 3-5 years, despite the profound changes in the structure and mechanisms of the economy and in the manifestations and forms of cyclic development (Hubner, 1994). In Poland, major crises occur approximately every 10 years: 1968-1970, 1980-1981, 1989-1990 and 2001-2002, (2012-…).

The authors propose a method of determining a model of forecasting crises and economic downturns using the gradient measurement (synthetic measure) and the modified logistic function (log-logistic function - based on logistic law (Kuznets, 1971)) which was tested on the Polish economy.

There is a view in the economic theory that each manufacturing activity is subject to the logistic growth law, whose forms are e.g. the law of decreasing income from farming or the law of a relative decrease in efficiency of expenditure. The above mentioned laws are based on experience and empirical research and lead to the conclusion that each manufacturing activity depends on the quantity of expenditures and the technological process used. In the case of manufacturing activity, after the beginning, which is characterized by a slow growth, the increase in expenditures causes a dynamic growth of effects to the maximum. From that moment on, the growth of effects is increasingly smaller and finally disappears altogether. After that period, a rapid decrease can happen in certain cases. Similar relations can be found in sciences concerning enterprises, where a limited growth of interest and the sale of a given product (product life cycle), or a limited growth of a market share are in question. In each case there are such phases of growth that can be identified (Kuznets, 1971; Metcalfe, 2001; Mar-Molinero, 1980).

The development of the economy and cycle phases can be identified owing to changes in GDP (Datta and Agarwal, 2004). According to e.g. Grodinsky (1953), when a new sector appears on the market, many enterprises try to enter it in its early and quick stage of development. The next stage is the time of domination of the strongest country economies and elimination of the weakest ones. A strong (rapid) increase is characteristic of that stage, although it is slower than in its early stage. According to Grodinsky's

[1] For example, the classic Juglar cycle lasts 6-10 years, a series of short Kitchin cycles last 4 years. The long Kondratieff cycle lasts 50-60 years (Diebolt, Doliger, 2005).

proposal, the first stage is a pioneering stage (i.e. after political transformations like in post-communist countries) and the second stage is an expansion stage. In the final stage, the economy is expected to stop growing and remain stable for some time.

In order to identify the stages of market and economy development, for example economic indicators such as GDP or stock market indices of prices are needed. It can be said that values of the above mentioned financial parameters are linked with markets. Depending on the cycle phase, the growth of those data is different. In the early stage, the value of the above mentioned data is not high and their growth rate is low. After some time the market or the economy enters the phase of crisis or enters the phase of intensive growth. At this stage exponential data growth such as GDP or indices appears. The growth in time is increasingly smaller and the market enters the stability stage. It can be noticed that if the market is stable, the shape of the index usually assumes the form of a logistic curve (Prescott, Hansen 2005; Ambler, Cardia, Zimmermann 2004; Comin, Gertler 2006).

Phases of the market and economy cycle can be isolated in all those examples. All of them have undergone the early and intensive growth stage. All of them have also undergone slight growth retardation.

Methodology

Warning signal criteria

The aim of a warning forecast is to signal "early enough" unfavourable changes in selected business activity areas, described by time series. A warning forecast is, by nature, a long-term forecast; its characteristic feature is the fact that it does not provide values of forecast variables but only a warning against a possibility that unfavourable changes might occur.

Warning forecasting consists in forecasting a decrease in business activity. On the one hand, enterprises should be recipients of such forecasts, especially when they are preparing strategic targets of their activity. At the same time they should thoroughly assess the financial rates.

The warning forecast is constructed for any time series whose correct trend is increasing. In practice, it rarely happens that financial quantities describing a financial activity constantly increase, the growth period is followed by a stability or a decrease period. The warning forecasting predicts the occurrence of a phase of decreasing values in the series. This is why the warning forecast is defined as follows by Siedlecki (2006):

"The warning forecast is a formulated assumption based on the information given by time series that in the next moment T_0 the state of the analysed financial phenomenon will be lower than in T_0 - 1 moment.

The warning forecast formulated in $T = n$ moment is true when time series terms meet the condition:

$$T_0 > n, \qquad (1)$$

Where, T - future real series value".

The warning forecast should be formed not on the basis of raw time series burdened with random errors, but on the basis of a smoothing function. The choice of the correct smoothing function $f(T)$ is significant in the warning forecast. In such a case the condition of the warning forecast is as follows:

$$f(T_0) - f(T_0 - 1) < 0. \qquad (2)$$

T_0 moment is called a warning forecast horizon. If the warning forecast horizon is too short, then the usability of the warning forecast is low because there is not enough time for repairing the process performance.

In practice, the analysis and the warning forecast are not usually based on one time series, but on a whole bundle of series which describe a selected fragment of the examined phenomenon.

In a company, the warning forecast horizon, which is the beginning of an unfavourable situation, is the change in the sign of second differences of a trend function of a selected series - signalling devices.

For any function in [1, n] interval, a sequence of signs of its first differences can be created. In a similar way, a sequence of signs of (t) function's second differences can be created. Let "+" mean positive (non-negative) and '-' means negative first (or second) difference of such f(t) function. A warning occurs when in a smoothing function's growth its second differences show the change of the sign from '+' into '-', whereas the warning disappears when the differences change the sign from '-' into '+'.

The warning is permanent in a time interval, starting from the point of inflection, via a maximum to a minimal point of the smoothing function. A very rough analysis of the graph permits assuming that in the near future a decreasing growth of the series will be maintained.

Use of gradient methods for the analysis of economic parameters

A gradient method based on the determination of the taxonomic distances of examined objects is interesting for the purposes of warning forecasting and constructing a synthetic indicator. The method may also apply respective economic parameters of the object model.

Let us assume that the matrix X comprises economic parameters, which are stimulants[1] x_{it}, where i (number of ratio) = 1,2, ..., m, t(time) =1,2, ..., n, and $x_{it} \in$ R:

$$X = \begin{vmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{vmatrix}$$

And assume that two data points (poles) are determined:
Top - a pattern of development

$$P = |p_{10}, \dots p_{m0}|, \qquad (3)$$

Bottom

$$Q = |q_{10}, \dots q_{m0}|, \qquad (4)$$

[1] Destimulants and nominants have to be converted into stimulants.

Where

$$p_{01} = \max_t x_{it}\ and q_{01} = \min_t x_{it}. \tag{5}$$

Then QP segment defines the axis of a set of synthetic indicators. PQ vector gradient can be treated as a linear programming criterion function Φ(t):

$$\Phi(\mathrm{t}) = \begin{vmatrix} p_{10} - q_{10} \\ \vdots \\ p_{m0} - q_{m0} \end{vmatrix}^T * \begin{vmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{vmatrix}. \tag{6}$$

The function Φ(t) shows the orthogonal projection of points. The values of this function are ordered synthetic indicators, where at any given time (point) t the following value is obtained:

$$\varphi(t) = (p_{i0} - q_{i0}) * x_{it}. \tag{7}$$

In the analysis of financial indicators, in order to eliminate the effect of their size on the arrangement, conversions should be made from matrix X to Z by the unitarization

$$z_{it} = \frac{x_{it} - \min(x_{it})}{\max(x_{it}) - \min(x_{it})}. \tag{8}$$

If the zero unitarization method is employed (formula 8), points P and Q will have the following form: $P = |1, \ldots 1|$, $Q = |0, \ldots 0|$. In this case:

$$\varphi(t) = \sum_{i=1}^{m} z_{it}. \tag{9}$$

And the measure of development (μ_t) will be:

$$\mu_t = \frac{\varphi(t)}{m}. \tag{10}$$

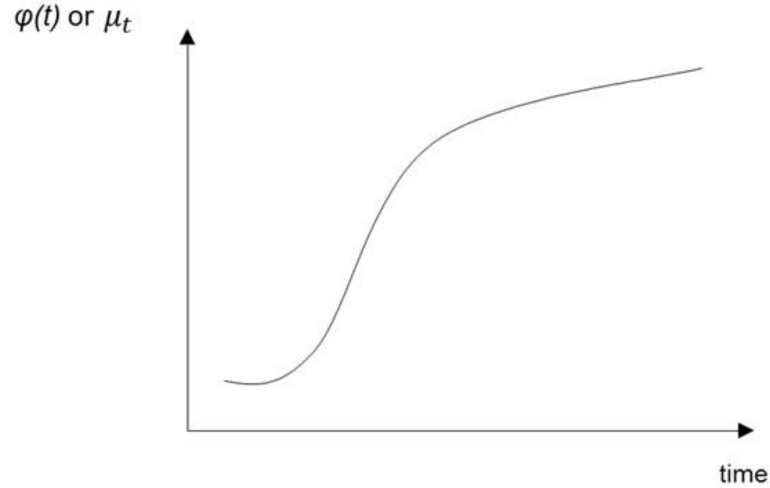
If μ_n (last measure) is close to 1, it denotes a good financial position. If it is closer to 0, the likelihood of financial distress is greater.

Assuming that the number of periods is greater than 4 and the zero unitarization method will be used (standardization has not been verified), then the interpretation of μ_t is as follows:

- $\mu t < 0.5$ distress area, high probability of economic crisis
- $0.5 < \mu t < 0.7$ grey area (warning signals - economic downturn)
- $\mu t > 0.7$ safe area

The warning in the company forecasts model is the state of the phenomena on at time t determined on the basis of the relevant economic ratios describing its development. So a measure of the development of a proper conduct should show the following relationship: $\varphi(1)<\varphi(2)<\ldots<\varphi(n)$, (or $\mu_1<\mu_2<\ldots< \mu_n$) which resembles a log-logistic (or logistic) curve, thus defining the development cycle of the economy (see Figure 1).

FIGURE 1. TRAJECTORY OF A HEALTHY ECONOMY



Source: own study based on Argenti (1976)

The analysis measures the development of the economy forecast. One way is to analyse the stability of increments and the volatility of economic parameters. So it can be argued that the warning signals decrease or increase beyond the allowable deviation from the trend function (logistic or log-logistic), designated by strategic bands.

Logistic and log-logistic functions

The logistic function is the mathematic expression of the logistic growth law. It has been put forward for the first time by P.F. Verhulst (Verhulst 1838). This function is most frequently used to describe economic or natural phenomena. It is the only solution of a differential equation called, in economics, the Robertson's, Prescott's, Kuznets' (Robertson 1923, Prescott 1922, Kuznets 1971) law:

$$\frac{dy}{dt} = \frac{-c}{a} y(a-y), \qquad (11)$$

On initial condition:

$$y(0) = \frac{a}{1+e^{b}}. \qquad (12)$$

And is expressed by the following formula:

$$f(t)=\frac{a}{1+e^{b-ct}}, \qquad (13)$$

Where, $a > 0$, $b > 0$, $c > 0$.

The logistic curve is a simple and universal, generally used and tolerably reliable tool for constructing distant economic forecasts (Davis 1941). At the same time it is also a method of measuring, observing and analysing the operation efficiency and effectiveness of technical devices characterized by great complexity or of large scale economic systems.

This function has two asymptotes $y = 0$ and $y = a$, determining the interval of variability of a given process. The upper determines the saturation level. The function has one inflection point separating the phase of accelerated growth from the phase of decreasing growth rate. Another important characteristic of the logistic function is its great flexibility, which allows very good approximation of the empirical data. This function is perfect for identifying the early stages of company development, i.e. from the origin phase to the intensive growth phase to the stagnation phase. However, it has one serious defect, being the horizontal asymptote limiting the growth.

It is a well-known fact that after the intensive growth of the values of financial data mentioned above either a collapse or a slow increase occurs.

It proves in many economic and financial cases that the logistic function does not work, which concerns mainly the "unlimited growth" phenomenon. As is known, quantities such as GDP, stock market indices, salaries in enterprises, sales or company value (it is well known that the aim of a company is to maximize its value in a long term) cannot be limited. If their value does not decrease rapidly after the intensive growth phase, then it is followed by a slow increase (its rate should fall to zero).

The way to eliminate the limited growth defect of the logistic function is to modify the function by introducing the ln(t) factor. The modified function is called the log-logistic function (logarithmic-logistic). The function was proposed by Z. Hellwig (Hellwig, Siedlecki 1989) and is expressed by the following formula:

$$f(t)=\frac{a\ln t}{1+e^{b-ct}}, \qquad (14)$$

Where $a > 0$, $b > 0$, $c > 0$.

When examining the function variability graph, its basic properties can be shown

$$\lim_{t\to\infty}\frac{a\ln t}{1+e^{b-ct}}=\infty, \qquad (15)$$

$$\lim_{t\to 0}\frac{a\ln t}{1+e^{b-ct}}=-\infty. \qquad (16)$$

And for $t_1 < t_2$

$$\frac{a\ln t_2}{1+e^{b-ct_2}} > \frac{a\ln t_1}{1+e^{b-ct_1}}, \qquad (17)$$

$$\frac{dy}{dt} > 0 \text{, for } t \geq 1. \qquad (18)$$

As can be seen, the log-logistic function is the function that grows constantly. It does not have extreme points and is always negative.

Log-logistic and logistic functions allow far extrapolation of time series. This is significant in forecasting of market phases of development using the logistic growth law, e.g. value of index or turnover.

Warning signal analysis based on the Polish Economy

Data selection

Warning signal tracing with use of an economy cycle of development should include: selection of suitable economic data, a suitable function for its forecast and variation and stability of the increment analysis.

One of the most important tasks in this case is to select the economic data and match a suitable function. The selection mainly depends on the economy of a given country (not on the sector), the period for which warning signals are supposed to be generated, and on a subjective appraisal of a person analysing those signals.

One of the manners of analysing financial warning signals, based on development of an economic cycle, is to analyse the increment stability and variation of selected economic and social parameters.

The analysis, based on the literature (e.g. Prescott, Hansen 2005; Guillen 2009) and own studies, selected 9 (from 21) indicators using the method of distribution convexity:

1. 0-unitarization method
2. Arrangement of data in ascending order
3. Determination of the frequency $\mathrm{w_{ij}} = \frac{1}{\mathrm{m}}$
4. Determination of the ratio ϑ

$$\vartheta_j = 1 - \sum_{i:x_{ij}\leq 0,5}^{n} , \quad \text{and } \ w_{ij}\,\vartheta_j < 0,5$$

Table 1 presents selected data for the Polish economy.

Nominants and destimulants have been converted into stimulants respectively:

- nominants:

$$x_{it} := -|x_{it} - median(x_i)|,$$

- destimulants

$$x_{it} := \max_t x_{it} - x_{it}.$$

TABLE 1. SELECTED DATA FROM 2000-2010

Variable	ϑ	Type
Revenues from total activity	0.363636	Stimulant
Total state budget revenue from personal income tax	0.363636	Stimulant
Relation of domestic debt to gross domestic product	0.181818	Destimulant
Total investment outlays (constant prices)	0.363636	Stimulant
Total population (as of 31 XII)	0.454545	Stimulant
Dwellings completed	0.181818	Stimulant
Gross Domestic Product (current prices)	0.454545	Stimulant
Gross agricultural output	0.454545	Stimulant
Total registered unemployed persons	0.454545	Destimulant

Source: Own study.

Log-logistic functions were estimated for the gradient measurement μ_t for Poland. The authors converted data by subtracting value of first observation to estimate the log-logistic function (without intercept). The parameters of the log-logistic function are presented in Table 2. The authors tested the stationarity of residuals of estimated functions using the KPSS test. For the authors' log-logistic model, they can reject the hypothesis of unit root with 0.05 level of significance.

TABLE 2. FIT OF THE LOG-LOGISTIC FUNCTION TO GRADIENT MEASUREMENT μT

	Polish economy
Log-logistic function parameters	a = 0.202223778 b = 16.43387148 c = 2.186562181
RMSE	0,026650535
KPSS test (H0: stationarity)	Test statistic = 0.158352 Critical values for significance levels: 10%, 5% and 1% are: 0.366 0.503 0.654
R^2 (2000-2010)	0.987044633

Source: Own study.

Warning signals

In this paper two methods of warning signal (possibility of crisis) estimation are used:

1. Critical values of gradient measures
2. Strategic bands.

The authors used data from period 2000-2010 for calculating gradient measurement μ_t and estimating the log-logistic function for determining of trajectory of economy development and phases of growth (second differences). Forecasts for years 2011 and 2012 are made in the next step.

In order to verify the forecasts and to designate warning signals (a decrease or an increase beyond the allowable deviation from the log-logistic trend function, designated by strategic bands) the authors normalized (and converted them into stimulants) real data form periods 2011 and 2012 using the following formula:

$$z_{it} = \frac{x_{it} - \min_{2000-2010}(x_{it})}{\max_{2000-2010}(x_{it}) - \min_{2000-2010}(x_{it})}.$$

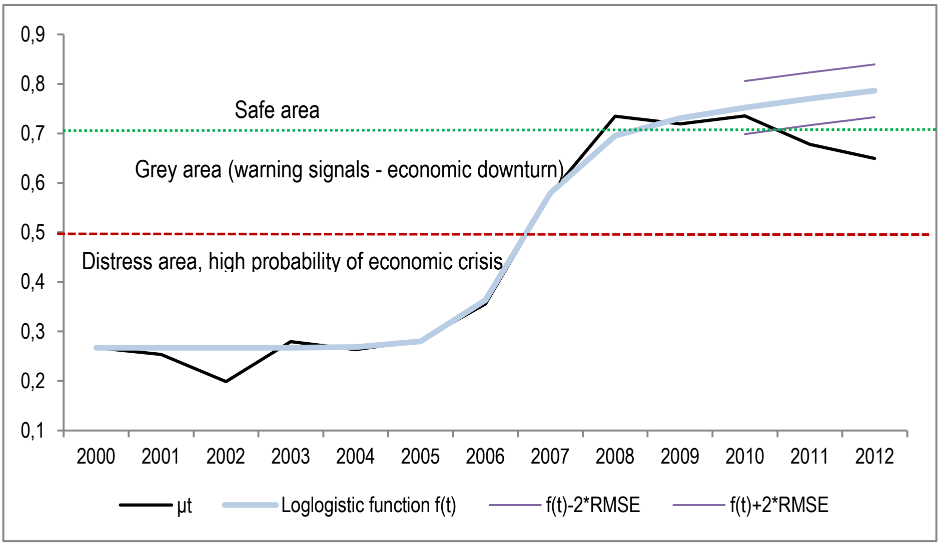
The parameters of gradient measurement μ_t are presented in Table 3 and Figure 2.

TABLE 3. GRADIENT MEASUREMENT μT OF POLISH ECONOMY

Year	μ_t	Log-logistic function	First differences	Second differences
2000	0.267284	0.267284	-	-
2001	0.25401	0.267285	1.36E-06	-
2002	0.198826	0.267295	1.27E-05	1.13E-05
2003	0.279487	0.267412	0.000117	0.000105
2004	0.263296	0.268607	0.001086	0.000969
2005	0.282082	0.279994	0.009846	0.00876
2006	0.355465	0.363514	0.074859	0.065012
2007	0.577744	0.579484	0.231351	0.156492
2008	0.734526	0.694952	0.129711	-0.10164
2009	0.718762	0.730893	0.020603	-0.10911
2010	0.735258	0.751957	0.002334	-0.01827
2011	0.677645	0.769763	0.000253	-0.00208
2012	0.64929	0.785974	2.73E-05	-0.00023

Source: Own study.

FIGURE 2. LOG-LOGISTIC FUNCTION ESTIMATION RESULTS



Source: Own study

The strategic bands are defined here as an acceptable deviation from the trend line determined by the log-logistic function (residuals are stationary). The root-mean-square error (RMSE) was used to determine the acceptable intervals.

$$RMSE = \left[\frac{1}{n} \sum_{j=1}^{k} (y_t - f(t))^2 \right]^{\frac{1}{2}}.$$

The strategic bands for the Polish economy have been constructed using the value of the determined log-logistic function as a point forecast, parameters of log-logistic function were estimated using data from 2000 to 2010.

The Polish economy generated a weak warning signal in 2008 because second differences of the log-logistic function show the change of the sign from '+' into '-' (see Table 3); it means that the economic growth slowed significantly. In 2011 and 2012 it was transformed into a strong signal because gradient measurement μ_t was less than 0.7 and exceeded the lower strategic band.

Conclusion

In this paper the authors successfully show how to use an interesting innovative measurement of economy development (gradient method) and the modified logistic function (log-logistic function) to determine the business cycle and to model a warning forecast.

The aim of a warning forecast is to signal "early enough" unfavourable changes in the economy, described by time series. A warning forecast is, by nature, a long-term forecast; its characteristic feature is the fact that it does not provide values of forecast variables but only a warning against the possibility that unfavourable changes might occur. The presented proposal to build a warning forecast (synthetic measurement) uses the taxonomic method of development based on gradient distance of development, which seems to be an interesting and efficient proposition for forecasting economic crises or downturns. The gradient measurement of development allows identification of the most favourable moment of the phases of the economy cycles with the use of the log-logistic function.

Certain concluding remarks can be derived from the presented examples:

- gradient measurement allows including many economic indicators in a synthetic way, which is very important in the modelling of economic phenomena
- gradient measurement properly informed about economic trouble in Poland
- log-logistic function is a very good tool for smoothing time series because it is monotonic and flexible
- log-logistic function allows far extrapolation of economic and financial time series,
- it is important in forecasting of economy and phases of financial development when using the logistic growth law
- the generated signals are clear and easy to interpret.

The presented proposal seems to be an interesting and efficient proposition for forecasting company financial difficulties and turning points in financial markets (Siedlecki, 2013; Siedlecki and Papla, 2012; Papla and Siedlecki, 2013).

References

Ambler, S., Cardia, E., Zimmermann, C., 2004. "International Business Cycles: What are the Facts?", Journal of Monetary Economics, Vol. 51, pp. 257-276, http://dx.doi.org/10.1016/j.jmoneco.2003.03.001

Berstein, J., 1989. The Analysis Forecasting of Long-Term Trends, Probus Books, Chicago

BIS, 2009. “The international financial crisis: timeline, impact and policy responses in Asia and the Pacific”, Bank for International Settlements (August)

Brunnermeier, M., 2009. “Deciphering the liquidity and credit crunch 2007-2008”, Journal of Economic Perspectives, Vol. 23, No. 1, pp. 77-100, http://dx.doi.org/10.1257/jep.23.1.77

Burns, A., Mitchell, W., 1946. Measuring Business Cycles, NBER, New York

Coffee, J.C., 2009. “What went wrong? An initial inquiry into the causes of the 2008 financial crisis”, Journal of Corporate Law Studies, Vol. 9, No. 1, pp. 21-22

Comin, D., Gertler M., 2006. “Medium-Term Business Cycles”, American Economic Review, Vol. 96, No. 3, pp. 523-551, http://dx.doi.org/10.1257/aer.96.3.523

Datta, A., Agarwal, S., 2004. “Telecommunications and economic growth: a panel data approach”, Applied Economics, Vol. 36, pp. 1649-1654, http://dx.doi.org/10.1080/0003684042000218552

Davis, H.T., 1941. The Analysis of Economic Time Series, Principia Press, Bloomington

Diebolt C., Doliger C., 2005, “Kuznets versus Kitchin, Juglar & Kondratieff Renewed Spectral Analysis of Comparative Growth of Per Capita GDP series in the OECD Countries in the Nineteenth and Twentieth Centuries”, Working Paper, EconPapers

Grodinsky, J., 1953. Investments, Part II, The Roland Press, New York

Guillen, M.F., 2009. The Global Economic & Financial Crisis: A Timeline. The Lauder Institute, Wharton University

Hellwig, Z., Siedlecki, J., 1989. “Loglogistic curve, properties and use for forecast of socioeconomic development” (in Polish), Prace Naukoznawcze i Prognostyczne 4

Hubner, D., 1994. Koniunktura gospodarcza, PWE, Warszawa

Kolb, R., 2010. Lessons from the Financial Crisis: Causes, Consequences, and Our Economic Future, Wiley, New York, http://dx.doi.org/10.1002/9781118266588

Kurach, R., Stelmach, J., 2009, “Analyzing the financial markets linkages - the subprime crisis case”, Research Papers of Wrocław University of Economics, No.59, pp. 260-268

Kuznets, S., 1971. Economic Growth of Nations, Belknap, Harvard, http://dx.doi.org/10.4159/harvard.9780674493490

Makridakis, S., Wheelwright, S.C., 1989. Forecasting Methods for Management, Wiley J. and Sons, New York

Mar-Molinero, C., 1980. “Tractors in Spain: a Logistic Analysis. The Journal of the Operational Research Society”, Vol. 31, No. 2, Feb, pp. 141-152

Metcalfe, J.S., 2001. “Industrial Growth and the Theory of Retardation: Precursors of an Adaptive Evolutionary Theory of Economic Change”, Working Paper

Mintz I., 1972. „Dating American Growth Cycles”, NBER, New York

Papla D., Siedlecki R., 2013. “Analysis of contagion in financial markets based on the gradient measurement of the growth in financial markets and conditional copula functions “, Working Paper, SSRN 2368456

Prescott, E.C., Hansen, G., 2005. “Capacity Constraints, Asymmetries, and the Business Cycle”, Review of Economic Dynamics 8, no. 4, pp. 850-865, http://dx.doi.org/10.1016/j.red.2005.08.001

Prescott, R., 1922. “Laws of Growth in Forecasting Demand”, Journal of the American Statistical Society, Vol. 18, pp. 471-479, http://dx.doi.org/10.1080/01621459.1922.10502490

Robertson, T.B., 1923. The Chemical Basis of Growth and Senescence, J.B. Lippincot Co., Philadelphia

Shiller, R.J., 2008. The subprime solution: how today's global financial crisis happened, and what to do about it, Princeton University Press, Princeton

Siedlecka, U., 1996. Warning forecasts in economy (in Polish), PWE, Warszawa

Siedlecki, R., 2006. Financial warning signals in company's cycle of life (in Polish), C.H. Beck, Warszawa

Siedlecki, R., 2011. "Method of determining warning signals based on a company's financial cycle of live", Economics, Management and Financial Markets, Vol. 6, No.1

Siedlecki, R., 2013. Forecasting Company Financial Distress Based on the Gradient Measurement of Development, Working Paper, SSRN 2374703

Siedlecki, R., Papla, D., 2012. "Method of Forecasting Stock Market Data Based on Law of Growth", Proceedings, 2nd Stochastic Modeling Techniques and Data Analysis International Conference, T. 4, pp. 137-147

Thieme, J., 2002. Tough times management (in Polish), Conference Board report, Warszawa.

Verhulst, P.F., 1838. "Notice sur loi que la population suit dans son accroisment", Correspondence Mathematique et Phisique, Vol. 10, pp. 113-121.