

# Non-linear corrections in market method of patent valuation

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Intellectual property rights are increasingly becoming an important asset of enterprises, so that an innovative business must carefully decide about the method of its valuation. The existing literature indicates three classical approaches to this issue: cost-based, income-based, and market-based methods, and a few more sophisticated ones such as: the option-based and patent citation methods, with their advantages and disadvantages. This paper proposes a novel methodology of non-linear corrections in the market model of patent valuation, when factors such as time to expiration, copying risk, or momentum in patent life cycle are taken into consideration. The proposed approach, based on evidence of the non-linear impact over time of the abovementioned factors on the value of patent, is anchored primarily in marketing science as well as in the theory and practice of accounting. This fine-tuning raises the accuracy and credibility of the market method of patent valuation.

**JEL Classifications:** G32, M4, O32, O34

**Keywords:** Patent valuation, market method, non-linear linking functions

## Introduction

Technological patents should be valued for the purpose of sales or licensing as well as because of the needs of taking them into account as assets of enterprises. There are three main methods of their valuation: cost-based, income-based, and market-based, and each has a number of advantages and disadvantages. There are also different recommendations for their use. The market method is popular and often the only one used in the case of unique goods. Information about past transactions for similar patents, though almost never identical due to the nature of these kinds of assets, helps to determine its current market price.

One of the drawbacks of the market-based method, beyond the difficulty of obtaining reliable and complete data from the market, is the way in which the value is adjusted in relation to similar market transactions. That which may distinguish patents is the added value obtained from the patented solution, the duration of patent protection, or the probability of breaking the patent etc. A comparison of the desired patent with the other transactions requires some adjustments due to the differences in the parameters.

A commonly used approach is linear correction, where the parameters are scaled proportionally, i.e. for example for the double length of patent protection, a double price is proposed. However, it distorts the actual impact of difference in parameters on the price, due to the nonlinearity of the real processes described with the chosen parameters. There are many models of non-linear changes in the marketing literature, including product life cycle, the cycle of market emotions, or the adoption cycle. Also, the accounting methods assume non-linearity, other than in linear models, for depreciation of assets, including so called degressive (with its subtypes: declining balance method or the method of the sum of digits of years) and progressive methods. Unfortunately, in the literature concerning the use of market methods, there is no guidance and recommendations on how to include this non-linearity in the processes of the adjustments

of the transaction price in the valuation of another patent. The purpose of this paper is to present the method of non-linear adjustment of different parameters in the market-based method of patent valuation, using linking functions that adjust the price in a universal way. The approach may be used independently of the input values.

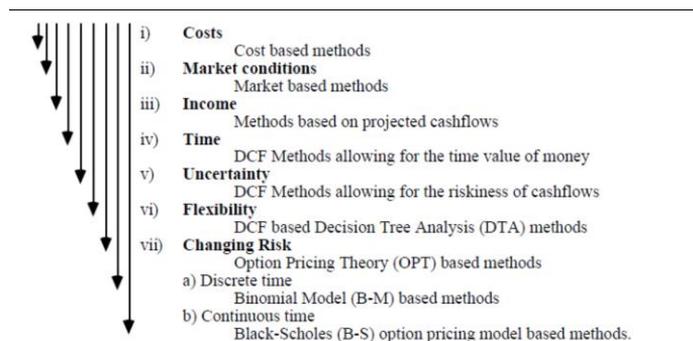
## Patent valuation - review of literature

The issue of the valuation of intangible assets is increasingly discussed in the literature. Patents, historically as one of the first and at the same time well-defined (in terms of legal and economic aspects) of industrial property rights, play an important role. However, it seems that along with the expansion of the catalog of objects in industrial property protection, the issue of the valuation of patents has granted a kind of *status quo*. Greater interest has shifted to the other intangible assets, the value of which over time is gradually included in the assets of enterprises, and only more or less sophisticated methods of their valuation are tried to be moved to patents valuation, through some form of analogy. For this reason, the two fairly significant limitations are emphasized in the existing methods of valuation of patents. Firstly, there is the use of quite simplified (somewhat historical) methods of valuation. The second is that the transfer of highly sophisticated, often complex and unclear for practitioners, models does not take into account the specific characteristics of patent protection.

It should also be underlined, that there are two specific, albeit contradictory, aspects that play an important role in research on the value of patents. On the one hand, patents, along with some others intangible assets are - also from an accounting point of view - closely identifiable assets of business (that are fully separable from other assets). On the other hand, intangible assets are merely a collection of acquired rights and privileges without the material form (as in physical assets), hence there is no explicit power to govern them - the access cannot be limited but only the legal restrictions may be applied. Those two aspects at the same time determine the need for strict valuation, but also show the limitations and difficulties of which one should be aware when treating such assets equally to other physical assets.

The literature defines three basic groups of patent valuation methods: cost-based, market-based, and income-based methods (e.g. Arthur Andersen, 1992; Reilly and Schweihs, 1998; Parr and Smith, 2000; Megantz, 2002). It may be said that historically, they have appeared in this order, and also this sequence represents their level of complexity. A more detailed classification was presented by Pitkethly (2002).

FIGURE 1. METHODS OF PATENT VALUATION BY PITKETHLY



Source: Pitkethly (2002), p.8.

From the above classification (Figure1), it is worth noting that the points iii) - vi), in fact, describe a broad group of income methods with additional factors like time, uncertainty, flexibility, which each affect the projected future revenues resulting from the protection of specific technological solutions. In addition, Figure1 also provides one example of more modern methods, which are tentatively applied to the valuation of patents, based on the valuation of financial instruments such as options. The abovementioned method uses binomial trees and modified Black-Scholes formulas. They explicitly emphasize the aspect of risk in estimations of future potential revenues from the use of patented technologies associated with different scenarios of market developments.

Despite the many advantages (Pugatch, 2005), including a more comprehensive look at the mechanism of functioning of the patent from the R&D stage up to its commercialization or abandonment, this approach may slightly impair the understanding of patents to treat it only as a conditional financial instrument requiring the strict assumptions about the distribution of parameters affecting the potential revenues. Even the use of the so-called concept of “real options”, that transfers the traditional models to the analysis of non-financial assets, causes a lot of problems (Pitkethly, 2002). In addition to the previously described group of methods, mention should also be given to the quite young, but still not very popular, method based on building econometric models which bind the value of the patent with variables such as the stock market value of companies using the patent, the patent renewals, the number of citations of the analyzed patent in the other patent specifications or technological solutions, the number of claims etc (Kamiyama et al., 2006; Rozek and Korenko, 2006). It is worth mentioning that in the literature, one may find many other patent valuation methods (which do not strictly fit the presented scheme) such as that described by Tönisson and Maicher (2012), which is based on consideration of 20 issues (steps) combining both quantitative and qualitative aspects.

The cost-based methods are considered to be the simplest method of valuation of patents and other intangibles. The value of a patent in this method is estimated by the identification of the amount of already incurred cost for the development of the new technology and getting its protection. The two methods of estimating these costs are usually considered: a method of reproduction cost and replacement cost. The first one focuses on identifying and updating the previously incurred costs, the second estimates the costs that would be incurred to re-produce the same, or as close as possible (in terms of economic effects) intangible assets. Cost-based methods, closely tied with the classical approach to accounting, are now considered archaic and not fully reflective of the actual market value of the patent. In addition to the difficulties in estimating and updating of those expenses, as well as difficulties with their isolation in the context of the valued assets, cost-based methods ignore probably the most important element of valuation which is the ability to generate and achieve future cash flows. Cost-based methods are oriented based on the factors that worked in the past.

Income-based methods, as one of the most popular, are applied to value many types of financial instruments and assets. By analogy and “out of habit”, they are also used in the valuation of intangible assets, including patents. Because of their widespread use, plenty of practices in this group have been developed, ranging from simple methods of discounting (only taking into account the time factor) to more sophisticated methods, incorporating additional factors. In many publications, both theoretical and practical, this group of methods is indicated as the most widely used and recommended. In general, all valuation methods belonging to this group are based on an attempt to estimate future cash flows generated as a result from holding or obtaining a patent for a specific technical solution. These flows are estimated directly, trying to identify the benefits generated by the product with patented solution, or indirectly, by comparing potential income with the revenue generated by similar products or products with similar functionality but without a protected solution.

The adjustment of estimated value can be provided on the basis of PV of future benefits, with the use of selected discount factor, also including the probability (uncertainty) of

previously assumed revenues from patented technologies. More complex methods can also be applied, mainly based on the construction and analysis of various market scenarios which are differently influencing on these future revenues. Despite its many advantages, including the well-established methods for estimating future cash flows, there are also disadvantages of this method. The first problem is the general risk of false assessment of the size and duration of the expected revenue, subsequently used in valuation of patents and other industrial property rights. The second problem is to reliably separate the cash flows which arise due to the utilization of protected technology, from the total cash flows associated with the product.

The market-based method is usually discussed quite briefly in the literature. It is probably the simplest method (in the sense of the idea), but attention is paid to its practical limitations. On the other hand, it is often considered to be potentially the best method of patent valuation. In fact, any good is worth as much as how much it is possible to exchange it for on the market, or put more formally, an equilibrium price is determined by the forces of supply and demand (Wirtz, 2012). Any other methods are only the search for the best approximations of this equilibrium price. In the market method, the valuation of patents and other intangible assets is carried out on the basis of comparison with the other recently made market transactions on assets with the most similar characteristics, preferably homogeneous ones. This leads to two very important limitations. Firstly, for the successful application of this method, the existence of an active market, in this case for patents, is necessary. The market should include assets with very similar characteristics to that which are to be subjected to valuation. This condition is true only for selected groups of patents, for example pharmaceutical patents. Moreover, the depth of the market (number of transactions) varies between countries, which will also limit the possibility of comparisons between countries. Even if such a market exists, a lot of the transactions are subject to trade secrets and thus the details of the transactions as well as their complete characteristics are not disclosed. However, not only the features of the protected technology affect the value of the transaction and therefore the patent, but many other factors can cause technologically similar patents to achieve different market values.

Pitkethly (2002) points at the modification of the market-based method, which may be used in place of that described above (see also Sullivan, 1994, Layne-Farrar et al., 2007). Considering the transactions on patents, more often than sales, the licensing is being offered. In such cases the owner of the patent can have the economic benefits of a protected technology without changing the holder of that right. The price of the license might be treated as the fee for use of the economic value of a patent, and at the same time as recompense for the depreciation of this asset. This depreciation is not primarily due to the lapse of time, but rather due to the permission for the presence of the protected technology in the offered products, which may result in an increase in the likelihood of its breaking, creating imitations or incentives for others to seek better technology, so that the patent will change its economic importance. Recommendation to refer the patent value to the amount of license fees, in the absence of a market for specific group of patents, is also not without limitations. In the case of exclusive licenses (possibly also with the commitment to reduce the use of protected technology by the owner) it can be assumed that the license fee directly indicates the market value of such a patent. In the case of non-exclusive licenses or open licenses, due to their formal, legal, and functional status, it might be misleading to directly take the size of royalties as the value of patent rights.

Today, one can observe that the market-based methods have their comeback in the valuation of patents. It is suggested that the barrier of faint markets can be overcome, when a number of patents that deal even with different but known characteristics are being observed. Schaffter and Zieger (2005) indicate that in patent valuation there is no need to operate with the other transactions as a whole, but it is essential to extract a set of indicators constructed on the basis of a broad set of data about past transactions. Using an appropriate combination of these indicators allows for a more precise estimation of value of a new patent. Wirtz (2012) explicitly calls this method 'valuation by analogy'. It

involves the transfer of a patent price on the basis of correction multipliers, obtained from the selected transactions (Mard et al., 2000). Those multipliers are a result of noticing common features as well as differences in past transactions and valued assets. In other words, it is simply a recommendation of the correction of transaction data available on pre-established rates, based on a specific set of factors. These factors cannot only be limited to economic (market) features, but must also arise from the very nature of the patent, i.e. to refer to the characteristics of technology in which the valued patent operates, its legal and formal status and other factors. Building an appropriate set of such factors and a methodology on how to use them appears to be a significant task at the moment in restoring market-based approaches to sufficient significance. It also seems that a simple linear correction method being applied to selected features cannot reliably fulfill its role in this case.

While there is a consensus that the cost-based methods are usually proposed as the third choice in the valuation of patents and other intellectual property rights (out of the three main groups discussed above), it is not clear that the most popular income-based method should be the first choice. Moreover, it is suggested that a market-based method may be chosen as the most appropriate one, both from the point of view of valuation for the accounting purposes and for the transactions on patents, but also in the context of the financial market: for example, the use of patents as collateral (Kamiyama et al., 2006; Pugatch, 2005). The disadvantages indicated above, namely the lack of a sufficiently large market for patents for finding a good comparable instruments, lack of reliable multipliers, or the lack of a full information on characteristics of the patents sold, means that the income-based method (although not free of major defects) emerges as the best solution. As proposed in this paper, the supplementation of market-based valuation method might become an important contribution in the extension of this model, further developing a mechanism known as the “analogy method”.

### **Product life cycle and non-linearity in patent valuation**

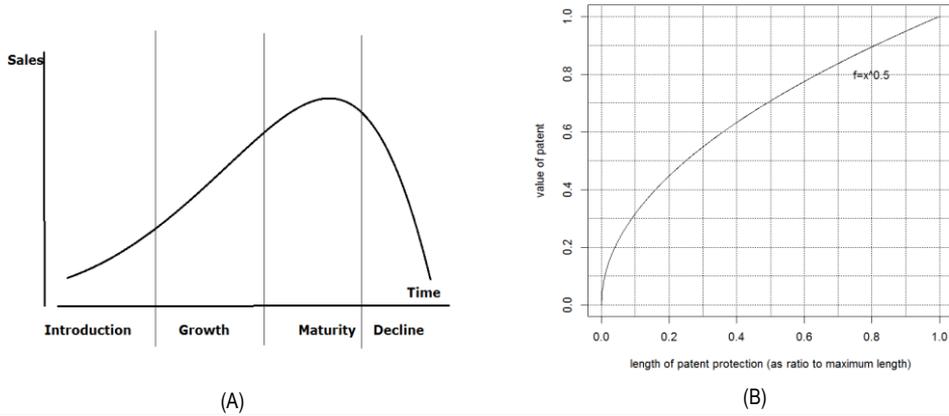
The primary objective of non-linear correction of the patent value is to differentiate the dynamics of changes in the value of the patent, depending on the value of the chosen parameter. There are two basic parameters to be adjusted: a) the probability of breaking the patent, and b) the remaining period of protection. A change in the value of a patent due to changes in these characteristics is in conjunction with the nonlinear processes that are well-known in marketing.

The theory of the product life cycle (see Figure 2a) implies that an increase in the value of sales is moderate in the introduction phase, the highest in the growth phase, reveals stagnation in the maturity stage, and is negative in the declining phase. It should be noted that these phases have an established chronology and it is possible to specify the length of their duration. The entire market life cycle differs between products depending on their type and the sector in which they are present, but it can be assumed that each of the four phases absorbs about 25% of the time throughout the product life cycle. With regards to the valuation, this non-linearity will be reflected in the patent value depending on the remaining time of patent protection. In the early years of patent protection, the value of a patent is highest due to rising income in the introduction and growth phase. In subsequent years, the sale of patent protected products is maintained at a constant level (maturity phase) and then decreases (decline phase), which translates to a faster loss of value of the patent than in previous years.

With regard to the time of patent protection, one can theoretically distinguish two types of adjustments:

- a) due to the remaining incomplete period of patent protection (e.g. 7 remaining years out of a maximum 20 years), appropriate for the transaction of granting a license,
- b) due to initially different durations of patent protection.

FIGURE 2. A) NON-LINEARITY CHANGES WITHIN THE PRODUCT LIFE CYCLE THEORY;  
 B) CHANGE IN VALUE VS. DURATION OF PATENT PROTECTION



Source: The value of patent and the length of patent protection is standardized to 1.

Due to existing international agreements in most countries, there are standardized periods of protection for industrial property rights: 20 years for a patent, 10 years for the protection of a utility model and trademark (noting that the right to a trade mark may be renewed every 10 years endlessly), 25 years from the registration of an (industrial) design or 10 years for the topography of an integrated circuit. This means that the corrections as in b) are, in practice, extremely rare.

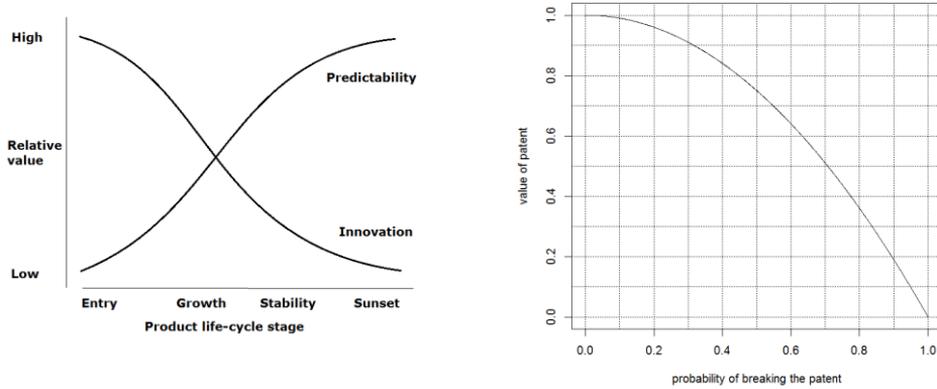
Figure2 shows the life cycle of the product, which in a statistical sense should be understood as a density function (see Figure 2a) and the cumulative distribution of the density function from the left panel (see Figure 2b). On Figure2 b, the X axis represents the scaled reminding time of patent protection, and the Y axis shows the scaled value of a patent. The cumulative distribution function represents the field under the (probability) curve, determining a percentage of the patent value (y) depending on the remaining term of protection of the patent (x). Full time protection ensures the full value (x = 100 %, y = 100 %), while a shorter period of protection decreases this value. The value decreases nonlinearly, slowly at first (e.g., from x = 0.6 to x = 0.9 ) and increasingly faster (e.g., below x = 0.1). This dynamic is consistent with the values obtained from the product life cycle plot (Figure 2a). For the adjustment described in a) when the patent has the full protection period t (t=1), it must be assumed that the patent is of the full value (V). When the protection period remains shorter (i.e. t=x, x<1) the difference in the valuation will be to the benefit of a patent with longer protection, V(t=1) > V(t=x). Then the correction factor C of patent value for a patent for less years will have a fractional value:

$$C = \frac{V(t=x)}{V(t=1)} < 1, \quad (1)$$

Another non-linear relationship is the extension of a product life cycle in the aspect of predictability and innovation. Highly innovative patents have the reduced chance of being copied and of losing their economic value. As shown on Figure 3a, in the phase of market entry, high innovativeness is accompanied by low predictability, in contrast to the decline phase. Innovativeness decreases nonlinearly, slowly at first, in the entry phase and growth

stage, and faster in the stability and inheritance phases. The slight decrease in level of innovation (low chance of breaking the patent) less than proportionally decreases the value of the asset. However, the high probability of breaking the patent, which in turn may harmonize with a significant reduction of innovativeness to the values close to 0, consequently may lead to a significant reduction in the value of a patent. Depending on the nature of the branch and the characteristics of a patent, the innovativeness function can pass from the concave to the convex (see Figure 3a) or be permanently concave (see Figure 3b) or convex.

FIGURE 3. THE INNOVATION FUNCTION



With regards to the valuation of assets, a greater chance of breaking the patent (resulting from a relatively low level of innovation or short term of protection remaining) reduces the value of a patent. When a patent has an expected probability  $p$  of breaking close to zero ( $p=0$ ), it must be assumed that the patent is of the full value  $V$ . When the probability increases (i.e.  $0 < p < 1$ ), the difference in the valuation will be in favor of the patent with lower probability of  $V(p=0) > V(0 < p < 1)$ . Then the correction factor of value of a patent  $C$  for a patent with a higher probability of breaking has the fractional value:

$$C = \frac{V(0 < p < 1)}{V(p=0)} < 1, \quad (2)$$

The two non-linear functions discussed above: product life cycle and the function of innovativeness, are not the only non-linear processes in marketing. One can give examples of other functions, including the non-monotonic and non-linear models, such as the cycle of market emotions, the ideas hype cycle, the death valley curve, the cycle of adoption, or the model of retaining customers over time. Such models may also be examined in the way described above.

### Financial accounting and the valuation of intangible assets

According to accounting rules, the intangible assets - including patents - belong to fixed assets. Taking them into account in financial statements will generally be subjected to the same rules, as tangible assets, including observation of the principle of a reliable and

accurate picture of the company by updating their value to the so-called “fair value”. In the case of assets that are present in market transactions, such a value is most often their market price; otherwise it shall be determined based on valuation techniques. At the same time, the book value of the assets should stay in reference to the actual valuation of the market, hence, over time there is a permanent need for the current valuation of fixed assets and making adjustments to their values.

In the balance sheet, the present value of assets, including intangible assets, can be determined using three mechanisms: a) amortization; b) impairment losses; c) revaluations (overestimations). The first and last mechanisms are particularly associated with the problems presented in this paper.

Although depreciation in modern accounting is considered as “the systematic timing of the financial burden (charges) with the value of fixed assets during their economically reasonable period of use” (Gmytrasiewicz and Karmanska, 2002), its source must be searched for in the observation of the different aspects (physical, economic, and legal and customary) of the systematic depreciation of such assets, and hence changes in their market or fair value. As a result, the current balance sheet value of fixed assets, which as indicated above should be in line with their market value, is reported as gross value adjusted by the accumulated depreciation. Leaving aside the two other methods, depreciation can be treated as a normal mechanism for evaluating the decrease in the price or value of fixed assets, including intangible assets such as patents, which are treated as business assets. Despite the fact that time is considered as a key factor in determining depreciation, for the precise identification of the expected period and the nature of economic usefulness the literature also gives other specific factors determining the expected period and the nature of the economic life (taking into account the specifics of intangible assets):

- the expected duration and extent of the use of intangible assets (such as patent rights);
- legal restrictions;
- expected obsolescence of technical and technological usefulness (resulting from changes in production processes, demand etc.);
- expected changes in the economic environment (e.g. in a given branch) , including the development and creation of new technological solutions;
- competitive actions, including the possibility of acquisition of a technology, its further development, as well as unauthorized copying, etc.
- parallel use of intangible assets and other assets.

The theory and practice of accounting has developed a number of methods of depreciation, for which the basis is the distribution over time of the abovementioned phenomena, for which the linear method of depreciation, degressive depreciation, and natural methods are the most often used. Particular importance in the context of the analyzed subject is the declining balance method (in various forms); because it is based on the observation of the non-linear impact at least some of the listed factors for value of assets.

The basis of the degressive method of depreciation is the observation that the usefulness of economic assets (and thus their value) decreases over time, although the rate of this change is faster at the beginning, thus in the initial period the impairment occurs faster. There are two basic patterns of accelerated depreciation: declining balance method and the method of the sum of the year’s digits. For the declining balance depreciation the general formula takes the form:

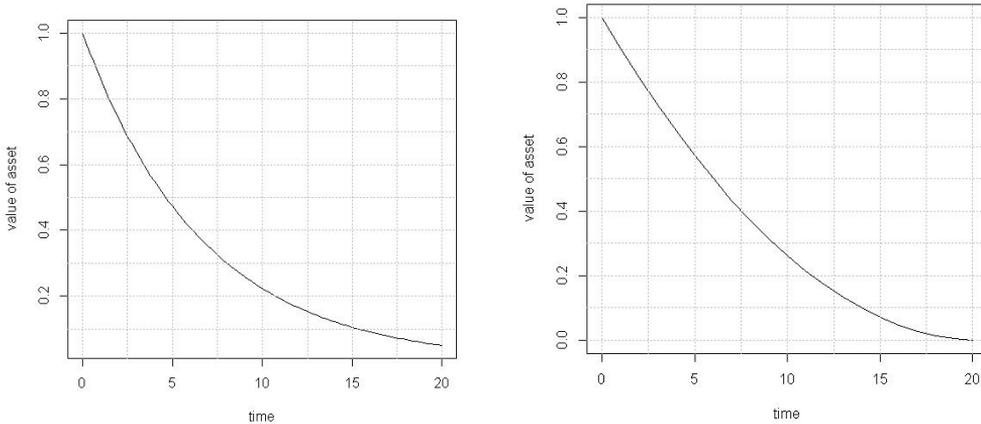
$$Aw_t = 1 - \sqrt[t]{\frac{Wr}{Wp}}, \quad (3)$$

where  $Aw_t$  is the annual rate of depreciation,  $Wr$  is the residual value,  $Wp$  is the initial value and  $t$  is the lifetime of the asset. In practice, the fixed rate of depreciation is assumed and then applied to the current (from the previous period) and not the initial value of depreciated goods, thus depreciation charges in future periods will get smaller and smaller. The method of sum of the year's digits assumes a different approach and directly points to a given rate of depreciation, which defines a periodic impairment calculated from the initial value, less its residual value, wherein the rate decreases over time. This model takes the form:

$$A\%_t = \frac{(T+(1-t))}{\frac{T(T+1)}{2}}, \quad (4)$$

where  $A\%_t$  is the depreciation rate for period  $t$ ,  $T$  is an expected duration of use, and  $t$  is the period for which depreciation is being calculated. Figure 4 shows an example of using both methods assuming a lifetime of asset equal to 20 years. The value of assets is normalized to the interval  $[0,1]$ .

FIGURE 4. THE VALUE OF ASSET ACCORDING TO A) DECLINE BALANCE METHOD AND B) THE SUM OF THE YEAR'S DIGITS



Source: Assumed 20 years of lifetime of asset and 5% residual value for decline balance method.

Similarly, not mentioned above, due to formal limitations and its rare use, the progressive method takes into account non-linear "wear" of the assets but, in contrast to the degressive method, it assumes the gradual acceleration of the depreciation process. It should also be noted that, permissible by law and often applied, the mechanism of change in depreciation methods for the assets already being used also indicates the attempt to take into account the different rates of decline in the value of the assets of enterprises. However, the admission of such discretionary adjustments seems to be a result of the inability to find a reliable complete model describing such changes.

Currently, in the context of the depreciation, accounting shows that the cost aspect is most important. However, primarily it was the impairment of fixed assets. Methods

described above for the determination of this periodic write-off of the initial value of assets, which are practically applied, indicate the non-linearity of the discussed process. Cited models and mechanisms can thus serve as a guide to develop a more general method of valuation of assets, including patents.

The use of the revaluation (overestimation) mechanism is the discretionary (although sometimes quite frequent) adjustment of value of assets, based on its current fair value, but according to International Accounting Standards (IAS 38), in relation to the value of intangible assets it can be made only when there is an active market for a given asset - in this case patents. Otherwise, the company is practically forced to adopt the gross book value as the original (historical) purchase price. With a few exceptions (e.g. in the area of pharmaceutical patents) the lack of a significant market is a typical situation and prevents the use of the revaluations. This tightening regulation seems to result from the lack of sufficient market-based valuation methods in the case of a small market for that type of asset. Thus it is not possible to reliably estimate the market value of an asset - such as patent rights - based on sporadically available similar assets on the market with non-homogenous characteristics. Despite the development of the intangible assets market, full knowledge on the nature and parameters of all transactions, as well as the emergence of an active market for all types of such assets, has not yet been achieved. Thus, there is a need, and even the necessity, for searching for new theoretical and practical solutions to this situation - in the future it would allow for the relaxation of the abovementioned limitations and allow for wider use of the mechanism for the overestimation of intangible assets. Proposed later in this paper is a solution of nonlinear adjustment of selected parameters in the valuation of patents which appears to meet this deficit.<sup>1</sup>

## Non-linear corrections

### **Market patent valuation - a basic approach**

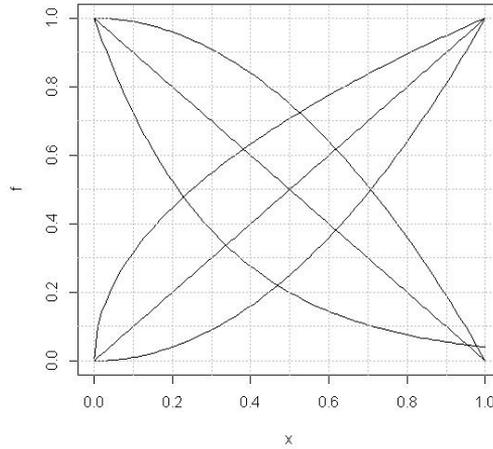
In the market patent valuation methodology, the basic assumption is that comparison with similar patents is justified. When the market price for a similar patent is known, it is related to the net benefit from using the patent. This net (after tax) benefit is calculated as additional gross margin on sales of products with the patented solution in relation to forecasted total sales. The ratio of *price-to-earnings* indicates payback period in years for the given investment in a patent. A crucial point in valuation is that the *price-to-earnings* ratio is assumed to be constant for compared patents. So the ratio for the patent which was sold is the same as for the patent to be valued. Thus, when predicted gross margin on sales and sales value are given for valued patent, its price can be calculated.

Difficulties arise when the parameters of both patents are different. Those can be, as mentioned above, the duration of patent protection, the estimated probability of breaking the patent, etc. Corrections are necessary and the linear solutions usually proposed do not fully reflect real differences in value as the market processes are not linear, as was explained previously.

Non-linear corrections are possible with the use of linking functions, scaled from 0 to 1 on x and y in the correction box. This gives a two-dimensional space (x,y). We assume that there are increasing functions from (0,0) to (1,1) and decreasing functions from (0,1) to (1,0) of three kinds: linear, convex, and concave (see Figure 5).

<sup>1</sup> An additional issue in which the proposed solution can also bring subsequent benefit is the need to estimate the initial value recognized in the accounting books at the time of acquisition of intangible assets acquired free of charge. In this case, accounting standards allow for using models because of the absence of market valuation.

FIGURE 5. CORRECTION BOX WITH LINKING FUNCTIONS



Source: Assumed 20 years of lifetime of asset and 5% residual value for decline balance method.

To obtain the desired shape of changes over time, according to references to real processes discussed earlier, some pre-defined functions can be used:

- a) linear and increasing:  $f_1 = x$
- b) concave and increasing:  $f_2 = \sqrt{x}$
- c) convex and increasing:  $f_3 = x^2$
- d) linear and decreasing:  $f_4 = 1 - x$
- e) concave and decreasing:  $f_5 = -x^2 + b$
- f) convex and decreasing:  $f_6 = a^x$

In the case of choosing functions e) and f), their parameters need to be calibrated. In f), function parameter  $a$  should be between 0.01 and 0.04, to reach approximately value  $y=0$  at  $x=1$  (not to differ much from 0). In function e) parameter  $b$  should be 1 to reach value  $y=1$  at  $x=0$ . The values of all six functions, calibrated with  $a=0.04$  in f) and  $b=1$  in e), were given in Table 1. In column *Ratio*,  $x$  values were given, as percentage (25%=0.25).

TABLE 1. VALUES OF LINKING FUNCTIONS

Ratio	Increasing functions			Decreasing functions		
	linear	Square root	parabola	linear	parabola	exponential
$x$	$f_1 = x$	$f_2 = \sqrt{x}$	$f_3 = x^2$	$f_4 = 1 - x$	$f_5 = -x^2 + b$	$f_6 = a^x$
0%	0.000	0.000	0.000	1.000	1.000	1.000
5%	0.050	0.224	0.003	0.950	0.998	0.851
10%	0.100	0.316	0.010	0.900	0.990	0.725
15%	0.150	0.387	0.023	0.850	0.978	0.617
20%	0.200	0.447	0.040	0.800	0.960	0.525
25%	0.250	0.500	0.063	0.750	0.938	0.447

TABLE 1. VALUES OF LINKING FUNCTIONS

Ratio	Increasing functions			Decreasing functions		
	linear	Square root	parabola	linear	parabola	exponential
30%	0.300	0.548	0.090	0.700	0.910	0.381
35%	0.350	0.592	0.123	0.650	0.878	0.324
40%	0.400	0.632	0.160	0.600	0.840	0.276
45%	0.450	0.671	0.203	0.550	0.798	0.235
50%	0.500	0.707	0.250	0.500	0.750	0.200
55%	0.550	0.742	0.303	0.450	0.698	0.170
60%	0.600	0.775	0.360	0.400	0.640	0.145
65%	0.650	0.806	0.423	0.350	0.578	0.123
70%	0.700	0.837	0.490	0.300	0.510	0.105
75%	0.750	0.866	0.563	0.250	0.438	0.089
80%	0.800	0.894	0.640	0.200	0.360	0.076
85%	0.850	0.922	0.723	0.150	0.278	0.065
90%	0.900	0.949	0.810	0.100	0.190	0.055
95%	0.950	0.975	0.903	0.050	0.098	0.047
100%	1.000	1.000	1.000	0.000	0.000	0.040

Note:  $f_5$  is calculated for  $b=1$ ;  $f_6$  is calculated for  $a=0.04$ .

The idea of the usage of linking functions is based on non-linear proportions of changes. In finding a correction coefficient, first the direction (increase vs. decrease) must be estimated by an evaluator and then the proper function needs to be chosen. Then the values of parameters for both patents (market and valued) are selected, in reference to the maximum value for a given parameter. When found, the ratio of two values of linking functions (see Table 1) is calculated and applied. The example presented below illustrates how to use practically linking functions.

#### **Practical example**

Let's assume that patent valuation follows the similar patent sales transaction. Knowing some parameters of the market transaction and the characteristics of the owned patent, the value of the transaction can be corrected to better suit the characteristics and the firm expectations.

Information on previous transactions is that a similar patent (in the same industry and with similar usage) was sold for €2 million euros. Additional gross margin on sales of products, resulting from having a patent, was 4% and annual sales of products with a patented solution is estimated to be €20 million euros. The remaining period of patent protection is 10 years from the total 20 years. The probability of "breaking" the patent (and the loss of economic value) is 20%. Tax rate is 20%.

The patent to be sold has the following characteristics: annual sale of products with a patented solution is forecasted for €12 million euros and additional gross margin from sales is 7%. The remaining period of patent protection is 14 years (out of 20 years) and the probability of breaking the patent is 40%.

For the patent which was sold, annual gross profit on sales is  $20\,000\,000 * 4\% = 800\,000$  euro, and after tax is  $800\,000 * 0.8 = 640\,000$  euro. As it was sold for 2 000 000 euros, the ratio of *price-to-earnings* is  $2\,000\,000 / 640\,000 = 3.125$  years - this is the payback period for the transaction. This ratio will be accepted as a constant for this kind of patent. For the newly priced patent, gross profit is  $12\,000\,000 * 7\% = 840\,000$  euro and after tax  $840\,000 * 0.8 = 672\,000$  euros. Assuming the same price-to-earnings ratio is equal to 3.125 years, the value of the patent before corrections would be  $3.125 * 672\,000 = 2\,100\,000$  euros.

The first correction in valuation is based on the remaining years of patent protection. The market patent had 10 years out of 20, which gives 50%. The priced patent has 14 years out

of 20 (70%). Firstly, the longer patent protection gives a higher value, and along with shortening the term of protection, this value is declining slowly, slower than the linear. With protection for 20 years, the first or second year after getting protection still guarantees high economic value, but for the last two years the patent is definitely worth less. Thus, function  $f_2 = \sqrt{x}$  seems to be the most appropriate. Secondly, longer patent protection should increase its value, so the correction is upward. The ratio of correction is then  $f_2(70\%)/f_2(50\%)=0.837/0.707=1.1832$ , and so it changes the priced patent value as follows:  $2\ 100\ 000 * 1.1832 = 2\ 484\ 720$  euros.

The second correction in valuation highlights the probability of breaking the patent and thus losing its economic value. For the previously sold patent probability was estimated as 20% and for the priced patent it is 40%. The higher probability of breaking the patent naturally decreases its value. Small probabilities have a slighter impact on the patent value than bigger ones, as the change from 2% do 4% of breaking the patent (100% increase) has less of an impact than a change from 50% to 100% (also 100% increase). This pattern of changes is well reflected by function  $f_5 = -x^2 + b$ . In this case, the correction ratio would be  $f_5(40\%)/f_5(20\%) = 0.84/0.96 = 0.8750$  and it changes the price to:  $2\ 484\ 720 * 0.875 = 2\ 174\ 130 \approx 2\ 175\ 000$  euros.

In this presented example, the initial value of newly priced patent would be 2 000 000 euros, but after estimation of the similar patent transaction this should be increased by 8.75% to 2 175 000 euros to fully cover recognized differences in the chosen parameters.

## Conclusion

This paper has developed fine-tuning corrections for the market-based methodology of patent valuation in a technical manner. An innovation that has not yet been proposed in the literature is the valuation adjustment in a non-linear way. Although most of the literature indicates a simplified linear approach to such corrections, it seems to be too far-reaching a simplification and has no substantive justification.

The proposal to use non-linear functions developed in the presented paper seems to be easy to implement and interpret. The linking functions and threshold values can be easily transferred to practical use in the valuation of intangible assets. The non-linearity in practice of depreciation for accounting purposes and widely adopted product life cycle models are important arguments in favor of the proposed method.

Of course, the proposed non-linear correction is justified only when the market-based method has strong grounds for its application. It is assumed that the conditions for the application of this model are the existence of at least partially comparable patents on the effectively functioning market, which have links to the newly valued asset. The comparability of patents must apply to some aspects such as importance for the consumer, business, growth prospects, legal protection, or the remaining duration of protection etc. It is also worth noting that the methods proposed in this paper of correction using linking functions can be easily transferred to other aspects than just those two described in the examples provided.

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