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Holographic description of extremal linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity

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Abstract – We show that extremal linear dilaton black holes in Einstein-Maxwell-dilaton-axion (EMDA) gravity have a hidden conformal symmetry. In this regard, we consider the wave equation of a massless scalar field propagating in this spacetime and find that in the "near region", the wave equation in the extremal limit can be written in terms of the $SL(2,R)$ quadratic Casimir operator. Moreover, we obtain the microscopic entropy of the extremal linear EMDA spacetimes also we calculate the correlation function of a near-region scalar field and find perfect agreement with the dual 2D CFT.

Introduction. – Recent investigations on the holographic dual descriptions for the black holes have achieved substantial success. In the context of the proposed Kerr/CFT correspondence [1], the microscopic entropy of the four-dimensional extremal Kerr black hole is calculated by studying the dual chiral conformal field theory (CFT) associated with the diffeomorphisms of the near-horizon geometry of the Kerr black hole. The main progresses are made essentially on the extremal and near-extremal limits in which the black-hole near-horizon geometries consist of a certain AdS structure and the central charges of the dual CFT can be obtained by analyzing the asymptotic symmetry following the method in [2] or by calculating the boundary stress tensor of the 2D effective action [3]. Recently, Castro, Maloney and Strominger [4] have given evidence that the physics of non-extremal Kerr black holes might be captured by a conformal field theory. They have pointed out that the wave equation for scalar fields in the Kerr spacetime has a certain conformal symmetry at low frequencies and in the region close to the horizon of the black hole, which they referred to as hidden conformal symmetry. This conformal symmetry is broken by the identification of the angular coordinates, from which those authors could read temperatures in a dual CFT and match the gravitational entropy of a non-extreme Kerr formula with a Cardy-type one. While the precise meaning of the initial paper is still not understood, it may be worth proving that the hidden conformal symmetry is a generic feature of black-hole spacetimes. Thus the initial computation of Castro *et al.* was repeated in some papers [5,6]. More recently the authors of [7] extended such investigations to the extremal case. The extreme limit is degenerate, so it is not completely straightforward to understand how hidden symmetry acts in this case. By introducing new conformal coordinates it was shown in [7] how to take the extreme limit, and the hidden-symmetry approach was applied to various extremal black holes, but the linear Dilaton black hole in Einstein-Maxwell-dilaton-axion (EMDA) gravity [8] was not addressed in that paper. The hidden-symmetry approach has been applied to the non-extremal linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity in [9]. The present paper fills the gap by combining the work of [7,9] to find the hidden symmetries of the extreme linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity. The EMDA black hole is a special example of four-charge spinning black holes with two non-zero pairwise equal charges [10]. So we consider the wave equation of a massless scalar field in the background of the extremal linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity and in the near region, we obtain the radial part of the wave equation. Then in new conformal coordinates we obtain the quadratic Casimir operator of $SL(2;R)$. The crucial observation is that these Casimir operators, when written in term of the coordinates φ, t, and r reduce to the radial equation of the massless scalar field. After

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that we obtain the macroscopic entropy of the extremal Einstein-Maxwell-dilaton-axion black hole. Moreover we find the absorption cross-section of a near-region scalar field matches with the microscopic cross-section in dual CFT.

Rotating linear dilaton black hole. – In this section, we give a brief review of the rotating-black-holes solution of Einstein-Maxwell-dilaton-axion gravity in four dimensions with the linear dilaton background [8,9],

$$\mathrm{d}s^2 = \frac{\Delta}{r_0 r}\mathrm{d}t^2 + r_0 r\left[\frac{\mathrm{d}r^2}{\Delta} + \mathrm{d}\theta^2 + \sin^2\theta(\mathrm{d}\varphi - \frac{a}{r_0 r}\mathrm{d}t)^2\right], \quad (1)$$

where $\Delta = (r^2 - 2Mr + a^2)$, and the background fields are given by

$$\begin{aligned} F &= \frac{1}{\sqrt{2}}\left[\frac{r^2 - a^2\cos^2\theta}{r_0 r^2}\mathrm{d}r\wedge\mathrm{d}t + a\sin 2\theta\mathrm{d}\theta\wedge(\mathrm{d}\varphi - \frac{a}{r_0 r}\mathrm{d}t)\right],\\ e^{-2\phi} &= \frac{r_0 r}{r^2 + a^2\cos^2\theta}, \qquad (2)\\ k &= -\frac{r_0 a\cos\theta}{r^2 + a^2\cos^2\theta}, \end{aligned}$$

where ϕ and k are the dilaton field and the axion field, respectively, F is the field strength of the Abelian vector field A.

The metric (1) was derived from the Kerr metric. When $r \to \infty$, the rotating linear dilaton black hole is asymptotic to the linear dilaton background. The event horizons of the black hole are given by the singularities of the metric function which are the real roots of $\Delta = 0$.

We now summarize the thermodynamics of a rotating linear dilaton black hole. The mass M appearing in the solution is no longer the physical mass, the mass $\widetilde{M}$ of the rotating linear dilaton black hole is

$$\widetilde{M} = \frac{M}{2}, \quad (3)$$

and the angular momentum J is given by [11]

$$J = \frac{ar_0}{2}. \quad (4)$$

The Hawking temperature T_H, the Bekenstein-Hawking entropy S_{BH} and the angular velocity Ω_H of the event horizon are given as

$$\begin{aligned} T_H &= \frac{r_+ - r_-}{4\pi r_0 r_+},\\ \bar{S}_{BH} &= \pi r_0 r_+, \qquad (5)\\ \Omega_H &= \frac{a}{r_0 r_+}, \end{aligned}$$

where $r_\pm = M \pm \sqrt{M^2 - a^2}$ are the locations of the outer and the inner event horizon, respectively.

Now we consider a bulk massless scalar field Φ propagating in the background of (1). The Klein-Gordon (KG) equation

$$\Box\Phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\Phi = 0, \quad (6)$$

can be simplified by assuming following form of the scalar field:

$$\Phi(t, r, \theta, \varphi) = \exp(-i\omega t + im\varphi)S(\theta)R(r). \quad (7)$$

The near region, which is the crucial region for demonstrating the origin of conformal structure, is defined by

$$r_0 \ll \frac{1}{\omega}. \quad (8)$$

In this background, the radial wave equation reduces to following equation:

$$\partial_r(\Delta\partial_r R) + \left[\frac{(r_0 r\omega - ma)^2}{\Delta} - l(l+1)\right]R = 0. \quad (9)$$

Let us consider the extreme linear EMDA black hole. In this case the Hawking temperature is vanishing, $\Delta = (r - r_+)^2, r_+ = M$ and $M = a$. In the low-frequency limit, we have the radial equation as

$$\left[\partial_r(\Delta\partial_r) + \frac{2(r_+ r_0\omega)(r_0\omega - m)}{r - r_+} + \frac{r_+^2(r_0\omega - m)^2}{(r - r_+)^2}\right]R(r) = l(l+1)R(r). \quad (10)$$

Hidden conformal symmetry . – In this section, we show that there exists a holographic 2D CFT description of the extreme linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity. In the literature [1,12], the standard treatment is to focus on the near-horizon geometry of the extremal black hole and analyze the asymptotic symmetry group to get the central charge of the dual CFT, while the temperature of the dual CFT could be read from the Hartle-Hawking vacuum. This kind of treatment has been applied to the study of the extremal linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity in [9]. Here we take a slightly different approach to find the holographic dual. Following [7] we now show that eq. (10) can be reproduced by the introduction of conformal coordinates. We will show that for the massless scalar field Φ, there exists a hidden $SL(2,R)$ conformal symmetry acting on the solution space. We can read out the left temperature of dual conformal field theory (in the extremal limit we have only the left sector). We introduce the conformal coordinates

$$\omega^+ = \frac{1}{2}\left(\alpha_1 t + \beta_1\varphi - \frac{\gamma_1}{r - r_+}\right), \quad (11)$$

$$\omega^- = \frac{1}{2}\left(\exp(2\pi T_L\varphi + 2n_L t) - \frac{2}{\gamma_1}\right), \quad (12)$$

$$y = \sqrt{\frac{\gamma_1}{2(r - r_+)}}\exp(\pi T_L\varphi + n_L t). \quad (13)$$

Now we can define the vector fields

$$H_1 = i\partial_+, \tag{14}$$

$$H_0 = i\left(\omega^+\partial_+ + \frac{1}{2}y\partial_y\right), \tag{15}$$

$$H_{-1} = i((\omega^+)^2\partial_+ + \omega^+ y\partial_y - y^2\partial_-), \tag{16}$$

and

$$\overline{H}_1 = i\partial_-, \tag{17}$$

$$\overline{H}_0 = i\left(\omega^-\partial_- + \frac{1}{2}y\partial_y\right), \tag{18}$$

$$\overline{H}_{-1} = i((\omega^-)^2\partial_- + \omega^- y\partial_y - y^2\partial_+). \tag{19}$$

Each of which satisfies the $SL(2,R)$ algebra

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0, \tag{20}$$

and

$$[\overline{H}_0, \overline{H}_{\pm 1}] = \mp i \overline{H}_{\pm 1}, \quad [\overline{H}_{-1}, \overline{H}_1] = -2i\overline{H}_0. \tag{21}$$

The quadratic Casimir operator is

$$\begin{aligned} H^2 &= \widetilde{H}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1}H_1) \\ &= \frac{1}{4}(y^2\partial_y^2 - y\partial_y) + y^2\partial_+\partial_-. \end{aligned} \tag{22}$$

The crucial observation is that these Casimir operators, when written in term of φ, t and r reduce to the radial equation

$$\begin{aligned} H^2 &= \partial_r(\Delta\partial_r) - \left(\frac{\gamma_1(2\pi T_L\partial_t - 2n_L\partial_\varphi)}{A(r-r_+)}\right)^2 \\ &\quad - \frac{2\gamma_1(2\pi T_L\partial_t - 2n_L\partial_\varphi)}{A(r-r_+)}(\beta_1\partial_t - \alpha_1\partial_\phi), \end{aligned} \tag{23}$$

where $A = 2\pi T_L\alpha_1 - 2n_L\beta_1$ and $\Delta = (r-r_+)^2$. Equation (10) can be rewritten as the $SL(2,R)$ Casimir operator (23) with the identification

$$\alpha_1 = 0, \quad \beta_1 = \frac{\gamma_1}{a}, \quad T_L = \frac{1}{2\pi}, \quad n_L = -\frac{1}{2r_0}. \tag{24}$$

The identification of T_L and n_L is consistent with the existing result [9].

We can directly calculate all the $SL(2,R)$ generators in terms of black-hole coordinates:

$$H_1 = \frac{2ia}{\gamma_1}(4a\partial_t + \partial_\varphi), \tag{25}$$

$$H_0 = i(-(r-a)\partial_r + 8a\phi\partial_t + 2a\varphi\partial_\varphi), \tag{26}$$

$$\begin{aligned} H_{-1} &= i\left(-\frac{\gamma_1\varphi}{a}(r-a)\partial_r + \frac{2M\gamma_1}{r-a}\partial_t \right. \\ &\quad \left. + \frac{a\gamma_1}{2}\left(\frac{\varphi}{a}\right)^2 + \frac{1}{(r-a)^2}\right)(4a\partial_t + \partial_\varphi), \end{aligned} \tag{27}$$

and

$$\begin{aligned} \overline{H}_1 &= \exp(-2\pi T_L\varphi - 2n_L t)\left((r-a)\partial_r - \left(\frac{a}{2} + \frac{4a^2}{r-a}\right)\partial_t \right. \\ &\quad \left. - \left(\frac{a}{r-a}\right)\partial_\varphi\right), \end{aligned} \tag{28}$$

$$\begin{aligned} \overline{H}_0 &= i\left(-\frac{2}{\gamma_1}\exp(-2\pi T_L\varphi - 2n_L t)(r-a)\partial_r \right. \\ &\quad -\frac{a}{2}\left(1 - \frac{2}{\gamma_1}\exp(-2\pi T_L\varphi - 2n_L t)\right)\partial_t \\ &\quad \left. -\frac{2a\exp(-2\pi T_L\varphi - 2n_L t)}{\gamma_1(r-a)}\right)(4a\partial_t + \partial_\varphi), \end{aligned} \tag{29}$$

$$\begin{aligned} \overline{H}_{-1} &= i\left[-\frac{1}{2}\left(\exp(2\pi T_L\varphi + 2n_L t) - \frac{4}{\gamma_1^2}\exp(-2\pi T_L\varphi \right.\right. \\ &\quad \left. -2n_L t)\right)(r-a)\partial_r - \left(\exp(2\pi T_L\varphi + 2n_L t) - \frac{4}{\gamma_1} + \frac{4}{\gamma_1^2} \right. \\ &\quad \left. \times \exp(-2\pi T_L\varphi - 2n_L t)\right)a\partial_t - \frac{a}{2(r-a)}\left(\exp(2\pi T_L\varphi \right. \\ &\quad \left.\left. +2n_L t) + \frac{4}{\gamma_1^2}\exp(-2\pi T_L\varphi - 2n_L t)\right)(4a\partial_t + \partial_\varphi)\right]. \end{aligned} \tag{30}$$

Actually there exists one degree of freedom γ_1 to define the vector fields, without affecting the form of the Casimir operator.

Real-time correlator. – The radial equation of the extremal linear EMDA black holes takes the form

$$H^2\Phi(r) = l(l+1)\Phi(r), \tag{31}$$

where l is a r-independent parameter contributing the conformal weight. With the ansatz (7), the radial equation can be written as

$$\left[\partial_r(\Delta\partial_r) + \frac{B}{r-r_+} + \frac{C^2}{(r-r_+)^2}\right]R(r) = l(l+1)R(r), \tag{32}$$

where

$$\begin{aligned} C &= (r_0 r_+\omega - ma), \\ B &= 2(2r_0\omega)(r_0 r_+\omega - ma). \end{aligned} \tag{33}$$

Introducing $z = \frac{-2iC}{r-r_+}$, we get the equation

$$\frac{d^2R}{dz^2} + \left(\frac{\frac{1}{4} - m_s^2}{z^2} + \frac{k}{2} - \frac{1}{4}\right)R(z) = 0, \tag{34}$$

where

$$k = ir_0\omega, \qquad m_s^2 = \frac{1}{4} + l(l+1). \tag{35}$$

This equation has the solution

$$R(z)=C_1R_+(z)+C_2R_-(z), \tag{36}$$

where

$$R_\pm(z)=\exp\left(-\frac{z}{2}\right)z^{\frac{1}{2}\pm m_s}F\left(\frac{1}{2}\pm m_s-k,1\pm 2m_s,z\right), \tag{37}$$

are two linearly independent solutions F is the Kummer function and can be expanded in two limits.

1) Near-horizon $r\to r_+$ so that $z\to\infty$:

$$F(\alpha.\gamma,z)\sim\frac{\Gamma(\gamma)}{\Gamma(\alpha)}\exp(-i\alpha\pi)z^{-\alpha}+\frac{\Gamma(\gamma)}{\Gamma(\alpha)}\exp(z)z^{\alpha-\gamma}. \tag{38}$$

2) When r goes asymptotically to infinity, $z\to 0$, $F\to 1$. In this case the solution has asymptotical behavior,

$$R\sim C_1r^{-h}+C_2r^{1-h}, \tag{39}$$

where h is the conformal weight

$$h=\frac{1}{2}+m_s=\frac{1}{2}+\sqrt{\frac{1}{4}+l(l+1)}, \tag{40}$$

and

$$C_1=-\frac{\Gamma(1-2m_s)}{\Gamma(\frac{1}{2}-m_s-k)}D,\quad C_2=\frac{\Gamma(1+2m_s)}{\Gamma(\frac{1}{2}+m_s-k)}D, \tag{41}$$

where D is a constant. The retarded Green function should read [13]

$$G_R\sim\frac{C_1}{C_2}\propto\frac{\Gamma(1-2h)\Gamma(h-k)}{\Gamma(2h-1)\Gamma(1-h-k)}. \tag{42}$$

Microscopic description. – In the extremal limit, the microscopic entropy comes to the left sector and for linear EMDA black holes $c_L=6r_0a$ [9], so that

$$S=\frac{\pi^2}{3}c_LT_L=\pi r_0a, \tag{43}$$

in agreement with [9]. We can determine the conjugate charge from the first law of thermodynamics. We begin with non-zero T_H, then take the limit to set it to zero. From the first law of thermodynamics,

$$\delta S=\frac{\delta\widetilde{M}-\Omega_H\delta J-\phi\delta Q}{T_H}. \tag{44}$$

In the limit $T_H\to 0$ we get [7]

$$\delta S=2\pi(r_0\delta\widetilde{M}), \tag{45}$$

we identify

$$\delta\widetilde{M}=\omega,\quad \delta E_L=\omega_L, \tag{46}$$

with

$$\omega_L=r_0\omega, \tag{47}$$

then

$$\delta S=\frac{\delta E_L}{T_L}=\frac{\omega_L}{T_L}. \tag{48}$$

Note that the identification (47) is the same as the one found in the study of non-extremal linear EMDA black holes [9]. The retarded Green function in extremal linear EMDA black holes should be rewritten as

$$G_R\sim\frac{\Gamma(1-2h)\Gamma(h-ir_0\omega)}{\Gamma(2h-1)\Gamma(1-h-ir_0\omega)}$$
$$\frac{\Gamma(1-2h)\Gamma\left(h-i\frac{\omega_L}{2\pi T_L}\right)}{\Gamma(2h-1)\Gamma\left(1-h-i\frac{\omega_L}{2\pi T_L}\right)}. \tag{49}$$

In a two-dimensional conformal field theory, the two-point functions of the primary operators are determined by the conformal invariance [7]. The retarded correlator gives the Euclidean correlator with this relation:

$$G_E(\omega_{L,E})=G_R(i\omega_{L,E}),\quad \omega_{L,E}>0. \tag{50}$$

At finite temperature, $\omega_{L,E}$ takes the discrete value of the Matsubara frequency

$$\omega_{L,E}=2\pi m_LT_L. \tag{51}$$

The momentum space Euclidean correlator is given by [7]

$$G_E\sim T_L^{2h_L-1}\exp\left(i\frac{\omega_{L,E}}{2T_L}\right)\Gamma\left(h_L+\frac{\omega_{L,E}}{2\pi T_L}\right)\times\Gamma\left(h_L-\frac{\omega_{L,E}}{2\pi T_L}\right). \tag{52}$$

The real-time correlator (49) is obviously in agreement with the CFT prediction [7].

Conclusion. – In this paper, we studied the hidden conformal symmetry of the extremal linear dilaton black hole in Einstein-Maxwell-dilaton-axion gravity. We solved the wave equation of a massless scalar field in the background of the extremal Einstein-Maxwell-dilaton-axion black hole and find that in the near region, the wave equation can be written in terms of the $SL(2,R)$ quadratic Casimir invariant. In other terms the quadratic Casimir invariant when written in terms of the coordinates φ, t and r reduces to the radial equation of the massless scalar field. In the paper [9] the authors have considered the non-extremal case and have shown that there exists a holographic 2D CFT description. In the extremal case we could not use the previous conformal coordinates as they were used in [9]. In the extremal case the Hawking temperature is vanishing. Therefore, only the left (right) temperature of the dual CFT is non-vanishing and the excitations of the other sector are suppressed. In the present paper we have done our job by using the new conformal coordinates, that were introduced in [7] for the first time. So according to these results there exists a dual CFT description of extremal linear dilaton black holes. Also the macroscopic entropy and the absorption cross-section of a near-region scalar field match precisely with those of the microscopic dual CFT.

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