Ranking the voice of customer with fuzzy DEMATEL and fuzzy AHP

Ali Reza Shahraki1 and Morteza Jamali Paghalah2*

1Department of Industrial Engineering, University of Sistan and Baluchestan, Zahedan, Iran
2Department of Industrial Engineering, Young Researchers Club, Islamic Azad University, Zahedan Branch, Zahedan, Iran

shahrakiar@hamoon.usb.ac.ir; morteza1362@gmail.com*

Abstract

Organizations and large companies consider “voice of customer” is an important factor for growth. The aim of our study is to achieve defined and suitable models of ranking the key criteria of the voice of customer. The method is based upon Fuzzy multiple criteria decision making (FMCDM). Fuzzy decision making trial and evaluation laboratory (Fuzzy DEMATAL) method, a useful group decision making tool, has been used to transform the complex interactions between the criteria of the problems of practical life into a visible structured model. The results indicate that in the presented case study, we could apply Fuzzy DEMATAL method to estimate the quantity of the effects of direct and indirect relations of elements with each other and promote the quality of relations and interrelations of the group.

Keywords: Fuzzy DEMATEL, Fuzzy AHP, FMCDM, Voice of Customer.

Introduction

Many different theories and methods of performance for conducting an evaluation have been applied in various organizations for many years. These approaches include ratio analysis, total production analysis, regression analysis, Delphi analysis, Balanced Scorecard, Analytic Hierarchical Process (AHP), Data Envelopment Analysis (DEA), Decision Making Trial and Evaluation Laboratory (DEMATEL), Fuzzy AHP (FAHP), Fuzzy DEMATEL, etc. Each method has its own basic concept, aim, advantages and disadvantages. Decision makers always like to know which option is the best of all alternatives.

In the category of cardinal information on the criteria or attribute of Multiple Criteria Decision Making (MCDM) methods, alternatives are ranked by their cardinal values of performance (Shih, 2008). Regardless of the type of MCDM task, two pivotal problems arise: how to compare an alternative? And how to evaluate them? The first problem is specifically important if the evaluation results of alternatives are represented by means of interval or fuzzy numbers (Sevastjanov and Figat, 2007). Decision-making is the most important and popular aspect in application of mathematical methods in various fields of human activities. In real world situations, decisions are almost always made on the basis of information at least partially fuzzy in nature (Vaidya and Kumar, 2006). Also Decision making is the process of defining the decision goals, gathering relevant criteria and possible alternatives, evaluating the alternatives for advantages and disadvantages, and selecting the optimal alternatives (Wu et al., 2009). However, in real life, the available information in a MCDM process is usually uncertain, vague, or imprecise, and the criteria are not necessarily independent. To tackle the vagueness of information and the essential fuzziness of human judgment or preference, Fuzzy Set Theory was proposed by Zadeh in 1965, and a decision-making method in a fuzzy environment was developed by Bellman and Zadeh (Kahraman, 2007). Fuzzy Set Theory was developed exactly based on the premise that the key elements in human thought are not numbers, but linguistic terms or labels of fuzzy sets. A fuzzy decision-making method under multiple criteria considerations is needed to integrate various linguistic assessments and weights to evaluate the location circumstances and determine the best selection (Chou, 2007).

MCDM may be considered as a complex and dynamic process including one management level and one engineering level. The management level defines the goals, and chooses the final “optimal” alternative. The multi-criteria nature of decisions is emphasized at this management level at which public officials called “decision-makers” have the power to accept or reject the solution proposed by the engineering level.

The DEMATEL technique was used to investigate and work on the complicated problem group. DEMATEL was developed based on the belief that the pioneering and proper use of scientific research methods could ameliorate comprehension of the specific problematic issues, the cluster of intertwined problems, and contribute to identification of practical solutions by hierarchical structure.

DEMATEL has been successfully applied in many situations such as marketing strategies, e-learning evaluation, controlling the systems and safety problems (Chen et al., 2009). Fuzzy DEMATEL method is used for solving and modeling some complex groups of decision-making problems such as strategic planning, e-learning evaluation and decision making in R&D projects (Coussement & Poel, 2008).

Voice of customers

Richins (1997) and Smith and Bolton (2002) emphasize that emotions differ when the context change; buying a pair of shoes may not generally raise the same kind of motions as do consumption where the amount of...
the monetary exchange is more considerable and may include risk (Roos et al., 2009). In systems of the real world, environmental planning and investment in sustainable development industries may essentially be conflicts analyses characterized by sociopolitical, environmental, and economic value judgments. Several strategies should be considered and evaluated in terms of many different criteria, resulting in a vast body of data that are often inaccurate or uncertain. However, in many areas such as manufacturing, engineering, medicine, meteorology, and human judgment, evaluation and decisions often employ natural language to express thinking and subjective preferences, but when using the word as a label for a set, there are some ranges involved within which the objects either do or do not belong to the set become fuzzy or vague.

It is now a leading firm strategy to develop a model from customer expectation. The service quality affects all leisure firm service activities. The performance is usually with multiple criteria for many customers’ expectations to judge for the best service quality performance. Improving service quality, increasing assessment and reliability occur while competition ever increases and tries to retain customers. Service quality conditions might influence a firm’s competitive advantage by retaining customer patronage and with this comes market share, and ultimate profitability. Service quality has been developed for several years. Service quality is measured to assess service performance, diagnose service problems, and manage service delivery. The criteria used for evaluation of service quality effectiveness are numerous, and influence one another Bell et al. (2005).

In the past, companies focused on selling services and products with little knowledge or strategy concerning the customers who bought the products. Today, business is evolving from ‘product-centered’ to a ‘customer-centered’ environment. Companies need to find ways to capture and enhance market share while reducing costs. Consequently, existing companies must reconsider the business relationships with their customers. Customer’s loyalty is defined as a consumer’s intent to stay with an organization. Since public services are not perceptible, fewer customers can directly evaluate services by comparing the quality of physical products (David et al., 2006). Customers and consumers constantly search for suppliers who provide much better goods and services. There are a lot of evidence and documents indicating that in today’s competitive world, discovering needs of customers and meeting their demands before the competitors is the basic term of enterprises to succeed. Thus, organizations and businesses are trying to achieve superior position than the other competitors by obtaining unique advantages. One of the most popular approaches to determine the quantity of meeting needs and demands by means of goods and services offered by organizations is assessment of customer’s consent.

In customer-oriented organizations, usually the customers are the most fundamental means to identify weak and strength points of organizations. During their relations, they can actually advise you to choose the method of communication with the customer, production and design of services. The organization can achieve strategies for new services by surveying its customers, analyzing their needs, and finding out its weak points in current services. Some authors believe that customers create different levels of making profits and all customers do not create a desired income for companies. Therefore, companies have been recommended to develop their relations with beneficial customers while cutting off relations with non-beneficial customers (Vasant et al., 2007).

Based on a study and an interview with some customers, researches could conclude with some important facts that determine satisfaction of customers with products and services of a company. Generally, the facts include: a). Stability of provider, b). Following schedule of goods delivery, c). Technical specification of products, d). Competitive prices, e). Credit policy of the company, f). Warranty and guarantee. Such items can put an effect on satisfactions with providers (Chakraborty et al., 2007).

Internet enables customers to easily express their problems with a product or a service. Consequently, customer complaint management and service recovery are going to become key drivers for improved customer relationships (Coussement & Poel, 2008). Further studied customer involvement in new product development, especially in the early steps of product conceptualization, plays an important role in product’s success (Chen & Yan, 2008). It is well recognized that voice of customer brings numerous benefits for organizations. They include customer loyalty, increase in satisfaction, product evaluation, mutually agreed problem solving, improved offerings, prevention of future problems, opportunity to redress the problem customer “venting” to the organization, rather than to others, and reduced likelihood of other negative consumer behaviors such as the exiting and/or negative word-of-mouth (Bove & Robertson, 2005).

Successful companies have to pursue customer-centred strategies in order to sustain a competitive advantage. Voice of Customer (VOC) analysis can play an important role in understanding customer requirements in a new product or service development. Moreover, it can provide value to customers and it can leave the customer with a favorable impression. The VOC analysis system can help determine what customers need and predict what they will need in the future. In turn, this can assist in the development of appropriate corporate strategies to meet the needs (Bae et al., 2005). However, different customers or experts have different attitudes toward the same requirement. To cope with this situation, it is proposed to use a group decision-making
technique to obtain the importance weights for customer requirements. Then, AHP or DEMATEL are proposed to be used in rating customer requirements, analyzing the sensitivity of the voice of customer in QFD (Lai et al., 2008).

**FMCDM and Fuzzy DEMATEL**

Human lives are the sum of their decisions - whether in business or in personal spheres. In daily lives, people often have to make decisions. "When decision is made" is as important as "what is decided". Everyday life and history are full of lessons that can help people recognize the critical moment. People learn by attempts and by tangible experiences. Deciding too quickly can be hazardous; delaying too long can mean missed opportunities. What people need is a systematic and comprehensive approach for decision making (Özdağoğlu & Özdağoğlu, 2007).

Fuzzy Set Theory introduced by Zadeh was developed for solving problems in which descriptions of activities, observations, and judgments are subjective, vague, and imprecise (Liu, 2009). Since Bellman and Zadeh (1970) developed the theory of decision behavior in a fuzzy environment, various relevant models were developed and they have been applied to different fields such as control engineering, artificial intelligence, management science, and MCDM among others. The concept of combining the fuzzy theory and MCDM is referred to as fuzzy MCDM (FMCDM).

A FMCDM model is used to assess alternatives versus selected criteria through a committee of decision makers, where suitability of alternatives versus criteria, and the importance weights of criteria, can be evaluated in linguistic values represented by fuzzy numbers. Numerous approaches have been proposed to solve fuzzy MCDM problems (Chua & Lin, 2009). Although the problem of obtaining well-defined criteria for a MCDM problem is well-known for more general results, it is often neglected in MCDM theory, methods, and applications.

Fuzzy set theory is very helpful to deal with the vagueness of human thoughts and language in making decisions. Decision-makers tend to give assessment according to their past experiences and knowledge, and also their estimations are often expressed in equivocal linguistic terms (Lin & Wu, 2008).

Many decisions are involving imprecision since goals, constraints, and possible actions are not precisely in description. In fuzzy logic, each number between 0 and 1 indicates a partial truth; whereas crisp sets correspond to binary logic (Anagnostopoulos et al., 2007). To deal with the vagueness of human thought and expression in making decisions, fuzzy set theory is very helpful. In particular, to tackle the ambiguities involved in the process of linguistic estimation, it is a beneficial way to convert these linguistic terms into fuzzy numbers (Tseng, 2009).

MCDM is a well-known branch of decision making. It is widely used in ranking one or more alternatives from a set of available alternatives with multiple attributes (Chou, 2010). MCDM techniques can help identify desired measures among a variety of alternatives through analyzing multiple criteria by which the strengths and weaknesses of various adaptation options could be evaluated. Thus, they could be adopted as evaluation tools to help identify the priorities of sustainable goals and to rank the desirability of adaptation options. Traditional MCDM models are based on the additive concept along with the independence assumption, but an individual criterion is not always completely independent. Multiple Attribute Decision Making (MADM), like MCDM, deals with the problem of helping the decision maker to choose the best alternative, according to several criteria. Alternatively, MCDM or MADM is the approach dealing with the ranking and selection of one or more vendors from a pool of providers. The MCDM provides an effective framework for vendor comparison based on the evaluation of multiple conflict criteria (Shyur & Shih, 2006).

However, in many cases, the judgments of decision making are often given as crisp values while crisp values are an inadequate reflection of the vagueness of the real world. Human judgment about preferences are often unclear and hard to estimate by exact numerical values, thus fuzzy logic is necessary for handling problems characterized by vagueness and imprecision. Hence, there has a need to extend the DEMATEL method with fuzzy logic for making better decisions in fuzzy environments (Wu, 2008). The DEMATEL method is used to construct interrelations between criteria to build an impact-relation map (Yang et al., 2008). FAHP or FMCDM analysis has been widely used to deal with decision-making (DM) problems involving multiple criteria evaluation or selection of alternatives (Hsieh et al., 2004).

**DEMATEL**

Complex evaluation environment can be divided into many criteria or subsystems to judge differences or measure scores of the divided criteria groups or subsystems more easily. The factor analysis method is commonly used to divide criteria into groups. The foundation of the DEMATEL method is graph theory. It allows decision-makers to analyze as well as solve visible problems (Chen & Chen, 2010).

The DEMATEL method has been successfully applied in many fields. Recently, there have been a lot of studies on fuzzy DEMATEL applications in different fields. DEMATEL method was developed by the Battelle Memorial Institute in Geneva. Those days, the DEMATEL method was used to study the world’s complicated problems, such as: race, hunger, environmental protection, energy, etc. In recent years, many scholars have broadly applied the DEMATEL method to solve problems in different fields. The DEMATEL method is an analytic technique of relationship structure. It can find the critical aspects or criteria of the complex structure system. The applicability of DEMATEL method is
widespread, ranging from analyzing world problematic decision-making to industrial planning. The original DEMATEL was aimed at the fragmented and antagonistic phenomena of world societies, and searched for integrated solutions. Digraphs are more useful than directionless graphs because digraphs can demonstrate the directed relationships of sub-systems. Moreover, digraph portrays a basic concept of contextual relations among the elements of the system, in which the numeral represents the strength of influence. The DEMATEL is based on digraphs, which can separate the involved factors into cause group and effect group. DEMATEL has been widely used to extract a problem structure of a complex problematic. By using DEMATEL, we could quantitatively extract interrelationship among multiple factors contained in the problematic. In this case, not only the direct influences but also the indirect influences among multiple factors are taken into account.

**Fuzzy DEMATEL**

DEMATEL method was presented in 1973 as a kind of structural modeling approach about a problem. It can clearly see the cause & effect relationship of criteria when measuring a problem (Chen et al., 2007). The decision-making involved in selecting appropriate management systems to create sustainable competitive advantages is a very important topic, which can be formulated as a MCDM problem. Applying the DEMATEL illustrates interrelations among criteria, finds central criteria to represent effectiveness of factors or aspects, and avoids “over fitting” for evaluation. Thus, non-additive methods, fuzzy measure, and fuzzy integral are used to calculate the dependent criteria weights and the satisfaction value of each factor or aspect for fitting with the patterns of human perception (Chen et al., 2007).

Although this DEMATEL method is a good technique for evaluating problems and making decisions, we decide the relationships of systems to be usually given by crisp values in establishing a structural model (Chiu et al., 2006). However, it is generally understood that human perceptions on decision factors are usually judged subjectively. The judgment in social science is always represented as exact numbers. In many practical cases, the human preference model is uncertain and might be reluctant or unable to assign exact numerical values to describe the preferences. The matrices or digraph portrays a contextual relation between the elements of the system, in which a numeral represents the strength of influence. Hence, the Fuzzy DEMATEL method can convert the relationship between the causes and effects of criteria into an intelligible structural model of the system. The Fuzzy DEMATEL method has been successfully applied in many fields (Coussement & Poel, 2008).

Evaluation criteria have the nature of causal relationships and usually comprise many complicated aspects. To gain a structural model dividing involved criteria into cause and effect groups, the Fuzzy DEMATEL method is an appropriate technique. To deal with the ambiguities of human assessments, the research discard the comparison scale used in crisp DEMATEL method but adopt the fuzzy linguistic scale used in the group decision making proposed (Coussement & Poel, 2008).

Different degrees of “influence” are expressed with five linguistic terms as “Strong”, “High”, “Low”, “No” and their corresponding positive triangular fuzzy numbers are shown in Table 1 (Shieh et al., 2010).

Table 1. The correspondence of linguistic terms and linguistic values (an example)

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Linguistic values</th>
</tr>
</thead>
<tbody>
<tr>
<td>No influence (No)</td>
<td>(0, 0, 0.25)</td>
</tr>
<tr>
<td>Low influence(L)</td>
<td>(0, 0.25, 0.5)</td>
</tr>
<tr>
<td>High influence(H)</td>
<td>(0.25, 0.5, 0.75)</td>
</tr>
<tr>
<td>Strongly influence(VH)</td>
<td>(0.5, 0.75, 1.0)</td>
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</tbody>
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Acquire the assessments of decision makers to measure the relationships between the critical success factors which are demonstrated by $C_{ij}$ ($i =1, 2, . . ., n$). The groups of the chosen experts were asked to make sets of pair wise comparisons in terms of linguistic terms. Hence fuzzy matrices $\tilde{N}_1, \tilde{N}_2, \ldots, \tilde{N}_5$, each corresponding to an expert and with triangular fuzzy numbers are obtained. Fuzzy matrix $\tilde{N}_1$ is called the initial direct relation fuzzy matrix of expert. Denote $\tilde{N}$ as:

$$\tilde{N}_{ij} = (l_{ij}, m_{ij}, u_{ij})$$

Without loss of generality, the $\tilde{N}_{ij}$ ($i = 1, 2, . . ., N$) Number $z = (0, 0, 0)$ when it is necessary.

Step 3. Establishing the structural model

The linear scale transformation is used here as a normalization formula to transform the criteria scales into comparable scales. Let

$$z = \left(\sum_{i=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} r_{ij}\right)$$

Then, the normalized direct-relation fuzzy matrix, denoted by $\tilde{N}$:

![Fig. 1. Fuzzy DEMATEL](image)

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Then, the normalized direct-relation fuzzy matrix, denoted by $\tilde{N}$:
\[ E = r^{-1} \otimes \mathbf{N} \]

Then \[ \mathbf{E} = \begin{bmatrix}
\tilde{e}_{11} & \tilde{e}_{12} & \ldots & \tilde{e}_{1n} \\
\tilde{e}_{21} & \tilde{e}_{22} & \ldots & \tilde{e}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{e}_{m1} & \tilde{e}_{m2} & \ldots & \tilde{e}_{mn}
\end{bmatrix}, \]

where \[ \lim_{n \to \infty} \mathbf{E}^k = [0]_{n \times n} \]. This assumption is well satisfied in practical cases.

Step 4. The total-relation matrix

The total-relation matrix \( \mathbf{T} \) can be acquired by using the following equation, in which the \( \mathbf{I} \) is denoted as the identity matrix.

\[
\mathbf{T} = \mathbf{E} + \mathbf{E}^2 + \ldots + \mathbf{E}^k = \mathbf{E} (\mathbf{I} + \mathbf{E} + \mathbf{E}^2 + \ldots + \mathbf{E}^{k-1}) = \mathbf{E} (\mathbf{I} + \mathbf{E} + \mathbf{E}^2 + \ldots + \mathbf{E}^{k-1}) (\mathbf{I} - \mathbf{E}) (\mathbf{I} - \mathbf{E})^{-1} = \mathbf{E} (\mathbf{I} - \mathbf{E})^{-1}
\]

When \[ \lim_{n \to \infty} \mathbf{E}^k = [0]_{n \times n} \], then \[ \mathbf{T} = \begin{bmatrix}
t_{11} & t_{12} & \ldots & t_{1n} \\
t_{21} & t_{22} & \ldots & t_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
t_{m1} & t_{m2} & \ldots & t_{mn}
\end{bmatrix} \]

where \[ t_{ij} = \left( \frac{e_{ij} + m_{ij} + r_{ij}}{m_{ij}} \right) \] (4)

\[
[t_i^r] = \tilde{E}_i \times (I - \tilde{E}_i)^{-1}
\]

\[
[m_i^r] = \tilde{E}_m \times (I - \tilde{E}_m)^{-1}
\]

\[
r_i^r = \tilde{E}_r \times (I - \tilde{E}_r)^{-1}
\]

Step 5. The sum of rows and columns

Produce a causal diagram. The sum of rows and the sum of columns are separately denoted as vector \( \mathbf{d}^m \) and vector \( \mathbf{r}^i \) through below formulas. In these equations, vector \( \mathbf{d}^m \) and vector \( \mathbf{r}^i \) denote the sum of rows and the sum of columns from the total-relation matrix \( \mathbf{T} \) respectively.

\[
\mathbf{T} = \begin{bmatrix}
t_{ij} \end{bmatrix}_{n \times n}, I,J \in \{1,2,3,...,n\}
\]

\[
d^m = (d^m_{ij})_{n \times 1} = \sum^n_{j=1} t_{ij} (x_{ij})_{n \times 1}, r^i = (r^i_{ij})_{n \times 1} = \sum^n_{j=1} t_{ij} (x_{ij})_{n \times 1}
\]

Step 6. As that of most fuzzy model, we had to covert the final fuzzy data into a crisp value. Here, we suggest the CFS (Converting Fuzzy data into Crisp Scores) method proposed by Opricovic and Tzeng for defuzzification. This method has the advantages of giving a greater crisp value with greater membership function and distinguishing two symmetrical triangular fuzzy numbers with the same mean. Let \( \mu(x) = \frac{1}{1 + e^{-x - u_k}} \); \( k = 1, 2...n \) be the positive triangular fuzzy number, and denote its representing crisp value. Computing \( L = \min (u_k); R = \max (u_k); k = 1, 2...n, \) and \( \Delta = R - L \), then

Step 7. We draw the causal diagram based on the calculations in step 6.

Step 8. Analyzing the results

Assume that \( \alpha_i^k \) denotes the row sum of \( i \)-th row of matrix \( \mathbf{T} \); then, \( \alpha_i^m \) shows the sum of influence dispatched from factor \( i \) to the other factors both directly and indirectly. Suppose \( r_j^i \) denotes the column sum of \( j \)-th column of matrix \( \mathbf{T} \). Then, \( r_j^i \) shows the sum of influence that factor \( j \) is receiving from the other factors (Chen et al., 2007).

The order of elements from column \( \alpha_i^m \) indicates hierarchy from influencing elements and the order of elements from column \( r_j^i \) \( r_j^m \) indicates hierarchy from influenced elements. The actual place of each element in the final hierarchy is determined by columns \( (\alpha_i^m + r_j^i) \) and \( (\alpha_i^m - r_j^i) \). If \( (\alpha_i^m - r_j^i) \) is a positive number, it is influencing and if it is negative, certainly, it is an influenced element. \( (\alpha_i^m + r_j^i) \)

Indicates the sum of density of an element along (longitude axis) regarding being either influencing or influenced. Final hierarchy is gained from the direct and indirect relations of \( (\alpha_i^m + r_j^i) \) and \( (\alpha_i^m - r_j^i) \) in the diagram.

**Fuzzy AHP**

The AHP was first proposed by Thomas L. Saaty in 1980. The AHP weighting is mainly determined by the decision makers who conduct the pair wise comparisons, so as to reveal the comparative importance between two criteria. If there are evaluation criteria, then to decide the decision making, the decision makers have to contest \( C(n, 2) = n(n-1)/2 \) pair wise comparisons.

The goal of MCDM method is to aid decision makers in integrating objective measurements with value judgments that are based not on individual opinions but on collective group ideas. Further, there are situations in which information is incomplete or imprecise or views that are subjective or endowed with linguistic characteristics creating a “fuzzy” decision making environment. The FMCMD approach is designed to minimize such adverse conditions and strengthen the partnership selection process (Chou, 2007).

Traditional evaluation methods usually take the minimum cost or the maximum benefit as their single index of measurement criteria, although these approaches may not be sufficient for the increasingly complex and diversified decision making environment. Thus, we utilize a FAHP to assess the sustainable development strategies for industry (Chiou et al., 2005). Fuzzy method weighs levels of criteria importance and
the determination of weights is the key point in comprehensive evaluation. The propriety of weights subsets will influence the results of the comprehensive evaluation.

AHP is a powerful method to solve complex decision problems. Any complex problem can be decomposed into several sub-problems using AHP in terms of hierarchical levels where each level represents a set of criteria or attributes relative to each sub-problem. The AHP method is a multi-criteria method of analysis based on an additive weighting process, in which several relevant attributes are represented through the relative importance.

In fuzzy MCDM problems, criteria or attribute values and the relative weights are usually characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are respectively defined as For brevity, triangular and trapezoidal fuzzy numbers are often denoted as (a, b, d) and (a, b, c, d). Human judgment of events may be significantly different based on individuals’ subjective perceptivity or personality, even when using the same words (Chiou et al., 2005). Fuzzy linguistic variables are extensions of numerical variables in the sense that they are able to represent the condition of an attribute at a given interval by taking fuzzy sets as their values. Triangular fuzzy numbers have been developed to appropriately express linguistic variables (Chiou et al., 2005).

AHP is widely used for multi-criteria decision making and has successfully been applied to many practical problems. If uncertainty (fuzziness) of human decision making is not taken into account, the results can be misleading. A commonality among terms of expression, such as “very likely”, “probably so”, “not very clear”, “rather dangerous” that are often heard in daily life, is that they all contain some degree of uncertainty. The concept of fuzziness, in traditional AHP, directly and without using fuzzy series has been taken into account. In fact, in this method, by using linguistic variables in Table 2, the concept of fuzziness is applied to determine pair comparison matrices. In this regard, we can refer to earlier models (Laarhoven & Pedrycz, 1983; Buckley, 1985; Chang, 1992; Lin, 2010).

A wide study in regard to these techniques can be observed in works of Kahraman (2004). Others used the fuzzy AHP procedure based on extent analysis method.

### Table 2. Numerical sum for preferences in pair comparisons

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Numerical sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference with full &amp; Absolute Importance</td>
<td>9</td>
</tr>
<tr>
<td>Preference with very strong importance</td>
<td>7</td>
</tr>
<tr>
<td>preference with strong importance</td>
<td>5</td>
</tr>
<tr>
<td>Preference with little importance</td>
<td>3</td>
</tr>
<tr>
<td>Preference with equal importance</td>
<td>1</td>
</tr>
<tr>
<td>For preferences between above linguistic terms</td>
<td>2, 4, 6, 8</td>
</tr>
</tbody>
</table>

and showed how it can be applied to selection problems. In this study, fuzzy AHP is described based on extent analysis method by Chang because this method has been simpler than other fuzzy AHP and similar to the method of classic AHP method.

**Extent Analysis Method of Chang**

If \( X = \{ x_1, x_2, ..., x_m \} \) is the set of objects and \( U = \{ u_1, u_2, ..., u_m \} \) is Wishes, then based on the extent analysis method by Chang, by considering one object, the extent analysis can be considered for every Wish \( g \). Therefore, there is the sum of “m” extent analysis for each object:

\[
\alpha (X) = \sum_{i=1}^{n} o_i \left[ M^1_{g_1} M^2_{g_2} \ldots M^m_{g_m} \right]
\]

The matrices of (Wish (W)) and (Object (O)) are shown above.

Where \( g_1 \) is the goal set \( i = 1, 2, 3, 4, 5 \ldots n \) and all the \( M^1_{g_1}, (i = 1, 2, 3, 4, 5 \ldots m) \) are Triangular Fuzzy Numbers (TFNs) (Fig. 2).

**Fig. 2. Triangle fuzzy number (Celik & Er, 2009).**

\[ \begin{array}{c}
\begin{array}{c}
\text{if}\ n = 2 - 4 & 6 - 8 \\
\end{array}
\end{array} \]

The steps of Chang's analysis can be given as follows:

*Step 1.* To obtain a fuzzy compound equation for each object:

If \( M^1_{g_1}, M^2_{g_2}, ..., M^m_{g_m} \) are the sums of \( i \) objected with respect to \( m \) Wishes, then the fuzzy compound equation of \( m \) Wishes for \( i \) Objects is defined as below:

\[
S_i = \sum_{j=1}^{m} M^j_{g_j} \bigotimes \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M^j_{g_i} \right]^{-1}
\]

If \( M^i_{g_i} = (u_{i,1}, u_{i,2}, u_{i,3}) \), then \( \sum_{j=1}^{m} M^j_{g_j} \) is defined by the fuzzy addition operation of \( m \) extent analysis as below:
Step 3: The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $S_k$ can be defined by:

$$V(S \geq S_k, \ldots, S_k) = \min_{i=1}^{n} V(S \geq S_i, \ldots, S_k)$$

and if

$$d_k(A_1) = \min V(S_k \geq S_k)$$

then the weight vector is given in equation 8 (It is not worthy that the obtained weights are fuzzy):

$$W' = (d_1(A_1), d_2(A_2), \ldots, d_n(A_n))$$

Where $A_i (i = 1, 2, 3, 4, 5, 6, \ldots, n)$ are n elements.

Step 4: Normalization of vector $W$ and obtaining weight vector of normalized weight of W.

$$W = (d_1(A_1), d_2(A_2), \ldots, d_n(A_n))$$

Algorithm of fuzzy AHP in the method of extent analysis of Chang.

The general process of algorithm of fuzzy AHP in the method of extent analysis of Chang is as below:

Step 1. Building up a hierarchy for the problem

Step 2. Determining pair comparison matrices and judgment operations, in traditional state (absolute), Table 2 is used for judgment operations; that is, the corresponding number is entered the pair comparison matrices by linguistic preferences.

But in the fuzzy state, we enter the sum of corresponding number with linguistic preferences in pair comparison matrices by triangular fuzzy numbers (Anagnostopoulos et al., 2007). Table 3 can be used in this regard. The fuzzy numbers given here are not equal to regular linguistic comparisons 1 to 9 but they are suitable for Fuzzy AHP and are used.

It is to be mentioned that all elements on the main diagonal of pair comparison matrices are equal to (1,1,1) and if the element of row i and column j of pair comparison matrix is equal to $[x]$, then element of row j and column i of this matrix is equal to:

$$M_{ij}^{-1} = \left( M_{ij}^{-1} \right) = \left( \frac{1}{m_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right)$$

Step 3. Computing relative weights of criteria and options

To compute the relative weight of the options with respect to each criterion and the relative weight of criteria with respect to object, we use the extent analysis method of Chang for each of pair comparison matrices. Therefore, a
The key criteria as follows:

- Price
- Special features of telecommunications
- Capacity of customers (quantity)
- Colprossesor
- Price
- Flexibility of the equipment in future

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Step 4. Computing the final weight of the options

The final weight of the options is obtained by modulation of relative weights. The key criteria as follows:

- Price
- Special features of telecommunications
- Capacity of customers (quantity)
- Colprossesor
- Price
- Flexibility of the equipment in future

Now we obtain preferences of $S_i$: 

\[
V(S_i \geq S_j) = \min_i V(S_i \geq S_j) V(S_i \geq S_j) V(S_i \geq S_j) V(S_i \geq S_j) V(S_i \geq S_j) V(S_i \geq S_j)
\]

We calculate normalization of fuzzy numbers:

\[
W = (0.0267, 0.0176, 0.0498, 0.0223, 0.01988, 0.0087)
\]
Table 4. Computing the final weight

<table>
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<tr>
<th>Object</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
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References


