INSOLVENCY PROBABILITY IN REINSURANCE TREATY: A CASE STUDY IN MALAYSIA

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Abstract: In developing countries such as Malaysia, the availability of reinsurance arrangements provides several advantages to the primary insurers such as keeping their risk exposures at prudent levels by having their large risk exposures reinsured by another company, meeting client requests for larger insurance coverage by having their limited financial sources supported by another company, and acquiring underwriting skills, experience and ability of handling complex claims by depending on another company for such services. This paper aims to model insurance claims and assess the insolvency probability of reinsurance treaties. Claims data was obtained from one of the leading insurers in Malaysia and R programming with actuar package is used to compute the probability of insolvency.

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Introduction

The volume of reinsurance business in the Malaysian general insurance industry may be observed from the amount of premiums ceded to companies abroad and within Malaysia. In 1965 and 1975 for instance, the amount of reinsurance premiums ceded to companies abroad were RM12 million and RM60 million, equivalent to 17% and 21% of written premiums respectively. The amount increased to RM296 million and RM1223 million each in 1985 and 1995, deteriorating to 24% and 27% of written premiums respectively, but decreased to RM957 million in 2005, showing an improved percentage of 10% of written premiums (Lee, 1997; BNM, 1995; BNM, 2005). Based on the proportions of written premiums, there was a marked deterioration in the 1980s and 1990s in terms of the domestic retention capacity of premiums compared to the 1960s and 1970s, due to the fact that Malaysia has never imposed restrictions on foreign exchange ouflows for reinsurance purposes. For most companies, their limited financial resources and small capability in underwriting skills and handling complex claims have enhanced their dependence upon outside reinsurers, leading to the issue of unsatisfactory domestic retention of premium (Lee, 1997). The level of retention capacity improved however in the 2000s, largely due to the continuous efforts taken by the regulatory bodies and industry players, especially in encouraging domestic insurers and reinsurers to absorb higher proportions of large risks.

Several studies focusing on reinsurance, deductible and policy limit have been carried out in the actuarial literature. Zhuang (2008) established optimal allocations of policy limits and deductibles with respect to the distortion of risk measures, Hua and Cheung (2008) applied equivalent utility premium principle and study the worst allocations of policy limits and deductibles, Dimitriyadis and Oney (2008) modeled loss distributions through Allianz tool pack, derived premiums at different levels of deductibles and computed ruin probabilities, and Wang (1996; 1998) introduced the method of Proportional Hazard Transform for pricing risks, excess-of-loss coverages, increased limits, risk portfolios and reinsurances treaties. This study aims to model insurance claims and assess insolvency probability of reinsurance treaties. Claims data was obtained from one of the leading insurers in Malaysia and R programming with actuar package is used to compute the insolvency probability.

Loss model

Several parametric distributions were fitted on the claims amount using maximum likelihood method. The best model, each for one-parameter, two-parameter and three-parameter distributions, was selected by choosing the largest value of log likelihood function and is shown in Table 1. The tests of Kolmogorov-Smirnov (K-S), Anderson-Darling (A-D) and Bayesian Schwarz Criterion (BSC) were carried out to select the best model (Klugman et al. 2008). Table 2 shows that the best model selected is Burr distribution.

Pricing of layers

As a developing country, insurance industry in Malaysia seldom has a single local insurer able to cover a single large risk, especially in non-life insurance business. In practice, the coverage of large risks is usually divided into several excess-of-loss layers shared and underwritten by several insurers and/or reinsurers. Therefore, the pricing of layers is very crucial, especially in the process of dividing risks and pricing risks fairly for each insurer and/or reinsurer.

Let N denotes the random variable for claim frequency. The expected claim frequency can be calculated as

$$E(N) = \sum_{k=0}^{\infty} S(k),\ k = 0,1,\ldots, \quad (1)$$

TABLE 1. BEST MODELS

Parametric distribution	# of parameters	Estimated parameters	Log likelihood
Exponential	1	$\lambda = 0.000025$	-2 207.0
Gamma	2	$\alpha = 1.4637$ $\theta = 26279.57$	-2 199.9
Burr	3	$\theta = 86{,}426.43$ $\gamma = 1.5169$ $\alpha = 3.7783$	-2 197.0

TABLE 2. RESULTS OF KOLMOGOROV-SMIRNOV (K-S), ANDERSON-DARLING (A-D) AND BAYESIAN SCHWARZ CRITERION (BSC) TESTS

Parametric distribution	# of parameters	K-S Test	A-D Test	BSC
Exponential	1	0.18655	389.31	-
Gamma	2	0.11098	384.68	-2205.16
Burr	3	0.09454	383.87	-2204.40

where $S(.)$ denotes the survival function. Under PH-Transform assumption, the expected claim frequency is equal to (Wang 1996; 1998),

$$H(N) = \sum_{k=0}^{\infty} [S(k)]^r , \quad (2)$$

where r denotes the index of ambiguity degree. Let X denotes the random variable for claim severity. The expected claim severity is,

$$E(X) = \int_0^{\infty} S(x)dx , \quad (3)$$

whereas under the PH-Transform assumption, the expected claim severity is (Wang 1996; 1998),

$$H(X) = \int_0^{\infty} [S(x)]^r dx \cdot \quad (4)$$

If the amount of loss follows Burr distribution with parameters (α, θ, γ), Wang (1996; 1998) showed that the calculation of $H(X)$ also follows Burr, but with parameters $(r\alpha, \theta, \gamma)$. By implementing the frequency and severity approach, the expected aggregate claims can be calculated as $E(S) = E(N)E(X)$ whereas under the assumption of PH-Transform, the expected aggregate claims is equal to $H(N)H(X)$. The same approach may also be implemented for calculating the price of a layer. The average loss or mean severity of a layer $(d, d+u]$ may be written as,

$$E(M) = \int_d^{d+u} S(x)\,dx , \quad (5)$$

whereas under the PH-Transform, the average loss of the same layer is (Wang 1996; 1998),

$$H(M) = \int_d^{d+u} [S(x)]^r \, dx , \quad (6)$$

where M is the random variable for the loss of a layer $(d, d+u]$,

$$M = \begin{cases} 0, & 0 \le X < d \\ X-d, & d \le X < d+u \\ u, & d+u \le X \end{cases} .$$

Therefore, the expected aggregate claims is $E(S) = E(N)E(M)$, whereas under the PH-Transform, the expected aggregate claims is $H(N)H(M)$.

Table 3 shows the mean severity, mean frequency, burning cost, loaded rate and relative loading under PH-Transform assumption for several excess-of-loss layers assuming N follows Poisson($\lambda = 6$) and X is distributed as Burr($\theta = 86{,}426.43$, $\gamma = 1.5169$, $\alpha = 3.7783$). The burning cost is calculated as $\frac{E(S)}{SEP}$, where SEP denotes the subject earned premium, assumed to be RM10 000 000. The loaded rate is calculated as $\frac{H(N)H(M)}{SEP}$, which can also be written in the function of burning cost, $\frac{H(N)H(M)}{SEP} = (1+\xi)\frac{E(S)}{SEP}$, where ξ denotes the relative loading. It is worth to note that the relative loading, ξ, under PH-Transform assumption increases as the excess-of-loss layer, $(d, d+u]$, increases.

Table 4 shows the premium and relative loading under several assumptions of PH-Transform ($r = 0.9$, $r = 0.8$, $r = 0.6$). It should be noted that the lower the r, the higher the premium, implying that the relative loading is higher when ambiguity increases. In addition, the premium is lower when the layer, $(d, d+u]$, increases.

Insolvency probability

If the claim frequency follows Poisson(λ), the aggregate claims, S, follows compound Poisson where the variance of aggregate claims can be calculated as $Var(S) = \lambda E(M^2)$. The distribution of aggregate claims, S, by applying Central Limit Theorem, may be estimated by the Normal distribution. Therefore, the insolvency probability, i.e. the probability of having the aggregate claims larger than the aggregate premiums, under PH-Transform assumption is,

$$\Pr(S > H(N)H(M)) = \Pr\left(Z > \frac{H(N)H(M) - E(S)}{\sqrt{Var(S)}} \right) \quad (7)$$

Table 5 shows the insolvency probability under several linear loading assumptions, i.e. premium=$(1+\xi)E(S)$, and several PH-Transform assumptions, i.e. premium= $H(N)H(M)$.

TABLE 3. MEAN SEVERITY, MEAN FREQUENCY, BURNING COST, LOADED RATE AND RELATIVE LOADING

Layer	$E(M)$	$H(M)$ $(r=0.9)$	$E(N)$	$H(N)$ $(r=0.9)$	Burning Cost	Loaded Rate	Relative Loading
$(100k,300k]$	1 652.40	2 509.38	6	6.247	0.0009914	0.0015676	1.58
$(300k,500k]$	30.10	70.84	6	6.247	0.0000181	0.0000443	2.45
$(500k,700k]$	2.91	8.75	6	6.247	0.0000017	0.0000055	3.13
$(700k,900k]$	0.56	2.00	6	6.247	0.0000003	0.0000012	3.71
$(100k,900k]$	1 685.97	2 590.97	6	6.247	0.0010116	0.0016185	1.60

TABLE 4. PREMIUM AND RELATIVE LOADING

Layer	Premium $(r=0.9)$	Relative loading	Premium $(r=0.8)$	Relative loading	Premium $(r=0.7)$	Relative loading
$(100k,300k]$	15 676.10	1.58	25 177.84	2.54	41 193.44	4.16
$(300k,500k]$	442.54	2.45	1 095.53	6.07	2 743.96	15.20
$(500k,700k]$	54.66	3.13	172.62	9.90	550.16	31.54
$(700k,900k]$	12.49	3.71	46.59	13.88	175.61	52.32
$(100k,900k]$	16 185.79	1.60	26 492.58	2.62	44 663.17	4.42

TABLE 5. INSOLVENCY PROBABILITY

Layer	Insolvency probability					
	Linear loading			PH-Transform		
	$\xi=0.1$	$\xi=0.15$	$\xi=0.2$	$r=0.9$	$r=0.8$	$r=0.7$
$(100k,300k]$	0.49	0.48	0.47	0.42	0.29	0.13
$(300k,500k]$	0.50	0.50	0.50	0.48	0.42	0.29
$(500k,700k]$	0.50	0.50	0.50	0.49	0.46	0.37
$(700k,900k]$	0.50	0.50	0.50	0.49	0.48	0.41
$(100k,900k]$	0.49	0.48	0.47	0.42	0.29	0.12

It is worth to note that the insolvency probability under PH-Transform assumption is lower than the linear loading assumption for all layers, and the difference is lower when the layer, $(d,d+u]$, increases. Therefore, PH-Transform assumption may be used as an alternative in reducing the insolvency probability of the excess-of-loss layers in reinsurance treaties.

Conclusion

In this paper, we have modeled the amount of insurance claims, selected the best model using the tests of Kolmogorov-Smirnov, Anderson-Darling and Bayesian Schwarz, priced several layers of $(d,d+u]$ using frequency and severity approach, and computed insolvency probability for several layers of $(d,d+u]$. PH-Transform assumption may be used as an alternative in reducing the insolvency probability of the excess-of-loss layers in reinsurance treaties.

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