FINANCIAL MARKETS

WITHDRAWAL STRATEGY FOR GUARANTEED LIFELONG WITHDRAWAL BENEFIT OPTION

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Abstract: In this paper we present a price model for the Guaranteed Lifelong Withdrawal Benefit, an option embedded in Variable Annuity policies, which gives the insured the possibility to withdraw from a fund a guaranteed amount annually, even if the account value has fallen below this amount. We calculate the No-arbitrage price of the contract if policyholders withdraw always the guaranteed amount or they surrender the product when the surrender value exceeds the value of future benefits.

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Introduction

Variable annuities (VA) were introduced in the 1970s in the United States (Sloane, 1970). In the USA the National Association of Variable Annuity Writers (NAVA, 2006) explains that "with a variable annuity, contract owners are able to chose from a wide range of investment options called enabling them to direct some assets into investment fund". For this reason, the VA contracts are defined the "close cousins of mutual fund, but they are formally classified as an insurance policy in addition to being registered as a security" (Milevsky and Salisbury, 2006). The VA, whose benefits are based on the performance of a underlying fund, are very attractive, because they provide a participation in the stock market and also a partial protection against the downside movements of the interest rates or the equity market. The typical VA is a unit-linked deferred annuity contract with one or more embed option. One of these is the Guaranteed Lifelong Withdrawal Benefit (GLWB), which enables the policyholder to receive a guaranteed amount until he is still alive. The first VA with a withdrawal benefit guaranteed for the life was introduced in the USA market in 2003. Since 2006 nine of ten VA products offered guaranteed living benefit; GLWB options represented the 35% of the whole market in early 2006 (Advantage Compendium, 2008).

In the light of the growing importance of this market, in this paper we present a pricing model and define a fair price for a GLWB in a market consistent manner. In order to achieve the first objective, our work use the standard No-arbitrage models of mathematical finance, in line with the tradition of Boyle and Schwartz (1997) that extend the Black-Scholes framework to insurance contract. The main difference is that for the option embedded in VA products the fee is deducted ongoing as fraction of asset, instead in the Black and Scholes approach the premium is paid up-front. The approach follows the recent actuarial literature on the valuation of VA products: Bauer et al. (2006); Chen et al. (2008), Coleman et al. (2006), Dai (2008), Holz et al. (2006), Milevsky and Posner (2001), Milevsky and Promislow (2001).

First, we adopt a static approach that assumes policyholders take a static strategy, i.e. the withdrawal amount is always equal to the guaranteed amount. In the dynamic approach, we describe the GLWB payoff if the policyholder surrenders the contract when the surrender value exceeds the value of future benefits.

Finally, we develop a numerical application to USA market in order to verify if the current US GLWB price is fair or the market seems to be underpriced or overpriced. Our conclusion is that the GLWB issued on the USA market are underpriced and this appears regardless of whether we take a static or dynamic approach. For example, our numerical results show that the No Arbitrage cost of a GLWB issued to a policyholder aged 60 would range between 79 and 145 basis points assuming a sub-account volatility in line with the average of the sub-account volatility for the universe of variable annuity products, while most products in the USA market only charge 50-70 basis points. Our results are in contrast to the common belief that the guarantees embedded in VA contracts are all overpriced (Clements (2004)); similar conclusions have been proposed for other options: Milevsky and Salisbury (2006) show that GMWB are underpriced on the USA market; also Chen et al. (2008) verify that the market fee are inadequate if the underlying risky asset follows a jump diffusion process.

The model

VA products with a GLWB option give the policyholder the possibility to annually withdrawal a certain percentage g of the single premium ω_0, that is invested in one or several mutual funds. The guarantee is lifelong: the maximum amount to be withdrawn is specified but the total amount is not limited and the insured can annually request a portion of the premium paid until he is still alive, even if the fund value drops to zero. Any remaining account value at the time of death is paid to the beneficiary as death benefit. The insurer charges a fee for this guarantee, which is usually a pre-specified annual percentage of the account value. If the policyholder withdraws the same amount each year the evolution of the fund is described by the following stochastic differential equation:

$$\begin{aligned} dW_t &= (\mu-\delta)W_t dt - Gdt + \sigma W_t dZ_t \\ W_0 &= \omega_0, \end{aligned} \qquad (1)$$

where W_t is the fund value at time t, μ is the drift rate of the fund process, δ is the insurance fee paid for the GLWB option, $G=g\omega_0$ is the guaranteed amount and Z is a standard Brownian motion.

Equation (1) holds while $W_t>0$. The discounted value at t=0 of the GLWB V_0 is the sum of the discounted values of the living benefits (*LB*) and death benefits (*DB*):

$$V_0 = LB_0 + DB_0, \quad (2)$$

where LB_0 is the discount value of a life annuity paying annually G and. In order to calculate DB_0 we observe that, since the maturity is stochastic and the time τ and $W\tau$ are independent, the following equation holds:

$$DB_0 = E_t\left\{E\left\{e^{-r\tau}DB_\tau \,|\, \tau=t\right\}\right\} = \int_0^{n-t} f(x)E_0(DB_t)dt \;, \quad (3)$$

where *f(x)* is the *pdf* of the future lifetime random variable for an individual aged *x* and *n* is the final age.

If we fix the date T, the death benefit can be calculated by Ito's lemma; the solution to equation (1) can be written as:

$$DB_T = e^{(\mu-\delta-\frac{\sigma^2}{2})T+\sigma Z_T} GT \max\left[\left(\frac{\omega_0}{GT} - \frac{1}{T}\int_0^T e^{-(\mu-\delta-\frac{\sigma^2}{2})t-\sigma Z_t} dt\right);0\right] \quad (4)$$

In the second approach, we describe the GLWB payoff if the policyholder assumes a dynamic strategy, according which he can lapse (i.e. withdraw more or less than the guaranteed amount from the found) and surrender the contract when he prefers. Generally VA contracts impose a penalty if the product is lapsed or surrendered prior to maturity. Supposing a proportional penalty charge *k* is applied on the portion of withdrawal above G, the net amount received by the policyholder is:

$$\ell(\gamma_t) = \begin{cases} \gamma_t & 0 \le \gamma_t \le G \\ G + (1-\kappa)(\gamma_t - G) & otherwise \end{cases}, \quad (5)$$

where γ_t denotes the discretionary withdrawal amount at the year t, if the insured is still alive. Following the notation in Holz at al (2007), any withdrawal strategy can be described by using a withdrawal vector $\gamma=(\gamma_1,..,\gamma_T)$. A full surrender strategy at time *t* is represented by allowing $\gamma_t=\infty$. The policyholder assumes a stochastic strategy if the decision whether and how much withdraw at time t depends on the account value and other information available at time t. Each stochastic strategy can be represented by a F_t-measurable process (X). Therefore, the value of the contract following the stochastic strategy (X) is given by:

$$V_0((X)) = \sum_{t=1}^{\omega-x}\left\{ {}_{t-1}p_x \cdot q_{x+t-1} E_Q\left[e^{-\int_0^t r_s ds}(\gamma(t,X)+DB(t,X)) \right]\right\}, \quad (6)$$

where ${}_{t-1}p_x \cdot q_{x+t-1}$ is the probability for a policyholder aged *x* to survive for *t-1* years and to die by the following year.

The rational policyholder maximizes the equation (6) choosing to withdraw the guaranteed amount or more or less than it. We analyze these two cases:

a. **$\gamma_t \le$ G:** for a GLWB withdrawing nothing or less than G can never be optimal. In fact, in a GLWB there is a lifelong guarantee and no adjustments are made for future guaranteed withdrawals. Hence, when the policyholder withdraws less than G, the future guarantees are the same, but their values are lower because W_t is greater. In addition, we have to consider that withdraw less than G involves a smaller living benefit and a greater D_t. However, due to the martingale property of the fund process and the fee deducted from the account value, the expected value of the additional death benefit is never greater than the withdrawal amount. So, the rational policyholder withdraws at least G.
b. **$\gamma_t = \infty$**: the policyholder surrenders the product when the surrender value exceeds the value of future benefits. Thus, we have to define a decision rule in order to establish the surrender time so that the zero value of the contract is maximized: for each possible scenario the rational policyholder would withdraw exactly the annual guaranteed amount until the value of the fund less the penalty exceeds the value of future benefits; then, he would surrender the contract.

Results

We apply our pricing model to GLWB options issued in the USA market, where the market fee ranges from 50 and 70 b.p. According to Morningstar Principia Pro, the average of the sub-account volatility for the universe of variable annuity products is 18%, the 25th percentile is 16% and the 90th percentile is 25%. We consider a policyholder aged 60 at the inception of the contract, the final age is n=110; in order to price the GLWB option we use the latest USA mortality table downloaded from the Human Mortality Database. We set ω_0=100 and the risk free rate *r*=5%. We calculate the fair insurance fee according the pricing model described in the previous section, under both static and dynamic approaches, in order to compare it with the current market fee. The results are obtained with Monte Carlo simulation. Once the interest rate, the volatility and the guaranteed rate have been fixed, we have searched the fair value of the fee with an iterative procedure: if the time-zero cost of the whole product turned out to be higher than ω_0 we increased the fee up to decrease the cost to ω_0; vice-versa, if the time-zero cost of the whole product turned out to be smaller than ω_0 we decreased the fee. In the dynamic approach we have constructed the simulated probability function of the optimal surrender time generating 10000 paths of evolution of the fund; for each path as soon as the value of the fund less the penalty exceeded the value of the future benefits we stopped the simulation. Table 1 compares the fair insurance fee under the static and dynamic pricing model for a policyholder aged 60 if g=5% and κ=10%:

TABLE 1. THE FAIR FEE UNDER STATIC AND DYNAMIC STRATEGY

Fair Fee	Static	Dynamic
σ=15%	54 b.p.	105 b.p.
σ=18%	79 b.p.	145 b.p.
σ =20%	96 b.p.	178 b.p.

Source: our computation

Conclusion

In this paper we have developed a pricing model for the GLWB option. First, we have taken a static approach that the withdrawal amount is always equal to the guaranteed amount. The opposite assumption we have considered is that policyholder surrenders the product at an optimal time, when the surrender value exceeds the value of future benefits. In this dynamic approach we have dealt with an optimal stopping problem and we have resolved it with Monte Carlo simulation.

Our paper fits in the actuarial literature on VA and investigate two aspects: the definition of a pricing model for the latest GLWB option, which takes in account both financial and actuarial aspects, and the verification of the fairness of the current GLWB price on the USA market. Our conclusion is that the GLWB issued on the USA market are underpriced and this appears regardless of whether we take a static or dynamic approach. On a practical side, our numerical results show that the No Arbitrage cost of a GLWB issued to a policyholder aged 60 would range between 79 and 145 basis points assuming a sub-account volatility in line with the average of the sub-account volatility for the universe of variable annuity products, while most products in the USA market only charge 50-70 basis points. This results indicate that the market fees are not sufficient to cover the market hedging cost of the guarantee. Of course, our pricing model does not allow for more sophisticated financial hypothesis, such as stochastic volatility or jumps in the fund process and term-structure effects, but as Milevsky and Salisbury (2006) we are confident that these considerations will only increases the price of the embedded option. The same effect would be obtained with the introduction of an actuarial model allowing for the longevity risk. So, we conclude by arguing that the current price of GLWB is not sustainable for insurers and the fees have to increase in order to avoid arbitrage opportunities.

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